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The Macroeconomic Impact of Immigration

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Abstract

This paper develops a macroeconomic model with skilled and unskilled labor to analyze the impacts of immigration on aggregate income and individual wage, labor employment and consumption in the host country. It is shown that immigrant inflows expand aggregate income but make some domestic workers worse off regardless of whether they are skilled or not, and independently of employment situations. However, the effect of labor productivity improvement, which is intended to offset such welfare losses, depends on the presence or absence of unemployment. For example, under full employment the productivity improvement of skilled labor enriches all people. In the presence of unemployment, it depresses aggregate demand and hurts the unskilled.

1. Introduction

Population aging due to longer life expectancy and lower fertility rates is a worldwide economic issue. According to *The World Population Prospects* by the United Nations, developed countries are expected to face sharp declines in the share of working-age population (see figure 1-1). Especially in Japan (see figure 1-2), a shortage of the labor force has been already serious, so that various policies including accepting more immigrants from abroad have been implemented to avoid economic contraction. In this paper, we build a tractable macroeconomic model with skilled and unskilled labor to analyze the impacts of immigration on aggregate income and individual wage, labor employment and consumption in the host country.

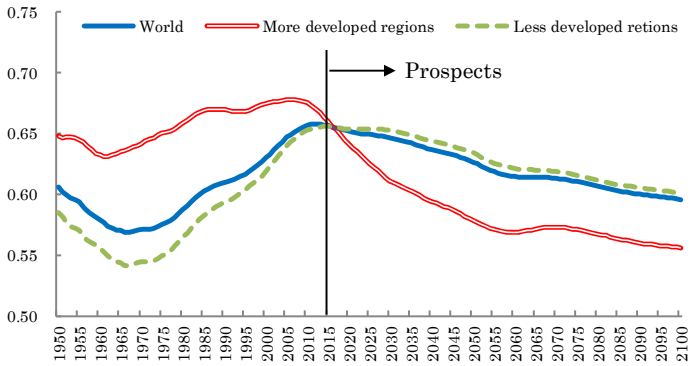

Figure 1-1. Share of working-age population

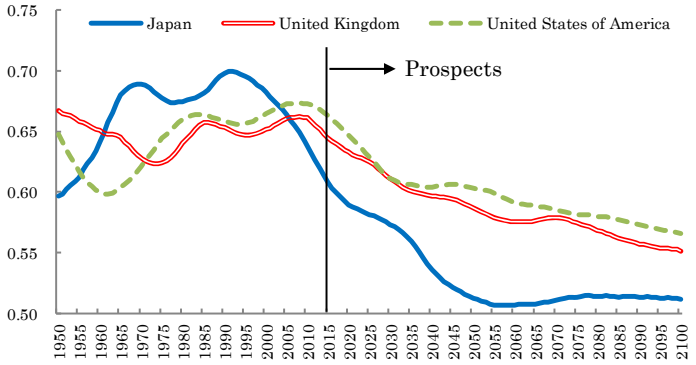

Figure 1-2. Share of working-age population

Source: Source: World Population Prospects: The 2015 Revision, United Nations.
Notes:
a) The share of working-age population is a ratio of population aged 15-64 years to total population in each region or county.
b) More developed regions comprise Europe, Northern America, Australia/New Zealand and Japan.
c) Less developed regions comprise all regions of Africa, Asia (except Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia.

Numerous studies explore the theoretical consequences of immigration, mainly assuming full employment of labor (e.g., Ben-Gad 2004, 2008, Moy and Yip 2006, Palivos 2009 and Palivos and Yip 2010).[1] However, it seems not to be the case for recent developed countries. In particular, Japan has experienced long-lasting stagnation with demand shortage and unemployment over the last two decades. After the global financial crisis in 2008, many European countries and the United States of America have followed the same path (see figure 2). Taking into account the possibility of demand shortage, this paper emphasizes that the presence or absence of unemployment is essential in discussing immigration policies since it drastically changes the implications. For example, in the presence of unemployment the job training, which is designed to improve the labor productivity of immigrants, takes employment away from domestic workers.

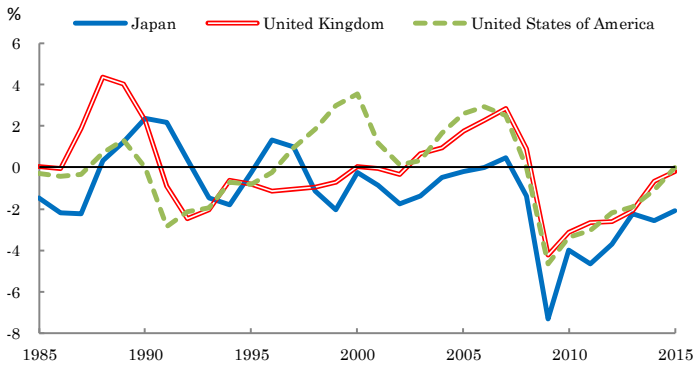

Figure 2. Output gap in percent of potential GDP

Source: Source: World Economic Outlook, International Monetary Fund.

[1] This paper focuses on investigating the theoretical implications of immigration. For a survey on empirical evidence, see Greenwood and McDowell (1986), Borjas (1994, 1995), Friedberg and Hunt (1995), and Card (2005).

There are several theoretical studies analyzing the relationship between immigration and unemployment due to labor market frictions—e.g., Ortega (2000), Liu (2010), and Chassamboulli and Palivos (2013, 2014) use a search and matching model of the labor market, whereas Carter (1999) and Kondoh (2004) employ an efficiency wage model. In such frameworks, supply-side inefficiencies are major causes of stagnation. This paper in contrast examines demand-side effects of immigration when demand shortage causes unemployment. It is shown that the difference in the propensity to consume between skilled and unskilled workers play a key role in understanding the effects of immigration policies.

The rest of the paper is organized as follows. In the next section, we present a benchmark model in which labor is fully employed. It shows that an increase of the labor force through accepting more immigrants hurts some domestic workers while raising aggregate income in the host country. To compensate such welfare losses arising from wage declines, the government should carry out the job training that improves labor productivity. It makes all people better off. Section 3 extends the analysis to the case of demand shortage. An increase of the labor force is shown to hurt some workers decreasing their labor employment. The result is seemingly similar to the full-employment case, but the mechanism behind them fairly differs. Actually, the productivity improvement of the skilled is harmful to the unskilled because it depresses aggregate demand by shifting labor demand from the unskilled to the skilled whose propensity to consume is lower. Therefore, the presence or absence of unemployment is crucial in considering the policy implications. The last section concludes the paper and proposes some future research directions.

2. The Structure of the Model

2.1. Firm and consumer behavior

Let us first specify the production structure of the economy. A firm uses two kinds of workers, skilled labor L^s and unskilled labor L^u, to produce a single final good, Y, according to the following constant-returns-to-scale technology:

$$\begin{aligned} Y &= F(\theta^s L^s, \theta^u L^u) \\ &= f\left(\frac{\theta^s L^s}{\theta^u L^u}\right)\theta^u L^u, \end{aligned} \tag{1}$$

where $\theta^i(>0)$ $(i = s, u)$ measures each labor productivity. (Hereafter, the superscript s (u) stands for a variable relating to the skilled (unskilled).) The marginal product of labor

is positive, $f'(\cdot) > 0$, and diminishing, $f''(\cdot) < 0$.

Given (1) and competitive markets, the wage rate w^i is set to equal the marginal product of each labor:[2]

$$w^s = \theta^s f'\left(\frac{\theta^s L^s}{\theta^u L^u}\right), \qquad w^u = \theta^u \left[f\left(\frac{\theta^s L^s}{\theta^u L^u}\right) - f'\left(\frac{\theta^s L^s}{\theta^u L^u}\right)\frac{\theta^s L^s}{\theta^u L^u}\right]. \tag{2}$$

This line depicted in figure 3 is downward sloping, indicating that labor demand falls as the own wage rate rises.[3]

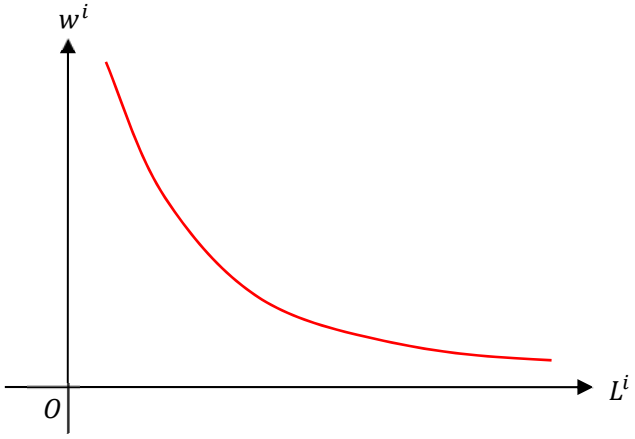

Figure 3. Labor demand curve.

Individuals' consumption depends only on their current disposable income.[4] In the absence of taxes and subsidies, they are

$$c^s = \sigma^s w^s l^s, \qquad c^u = \sigma^u w^u l^u, \tag{3}$$

$$\text{where} \qquad L^s = N^s l^s, \qquad L^u = N^u l^u. \tag{4}$$

c^i, l^i, and N^i denote consumption per capita, realized labor employment per capita, and the population of each worker, respectively. Workers are willing to supply one unit of labor inelastically but may be underemployed in the presence of demand shortage. Assume that the propensity to consume of skilled workers, σ^s, is smaller than that of unskilled workers, σ^u:

$$0 < \sigma^s < \sigma^u < 1.$$

In fact, figure 4 indicates that the poor income group consumes a higher proportion of their income than the rich.

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[2] The optimal conditions (2) are derived by maximizing the firm's profit, $Y - w^s L^s - w^u L^u$, subject to (1). See Mankiw (2015, chapter 3)

[3] This property is formally shown by differentiating (2) with respect to L^i:

$$\frac{\partial w^s}{\partial L^s} = f''\left(\frac{\theta^s L^s}{\theta^u L^u}\right)\frac{(\theta^s)^2}{\theta^u L^u} < 0, \qquad \frac{\partial w^u}{\partial L^u} = f''\left(\frac{\theta^s L^s}{\theta^u L^u}\right)\frac{(\theta^s L^s)^2}{\theta^u (L^u)^3} < 0.$$

[4] Although this assumption, called the Keynesian consumption function, is made for analytical simplicity, it is controversial. See Mankiw (2015, chapter 16) and Ono (2011) for the criticism against this consumption function.

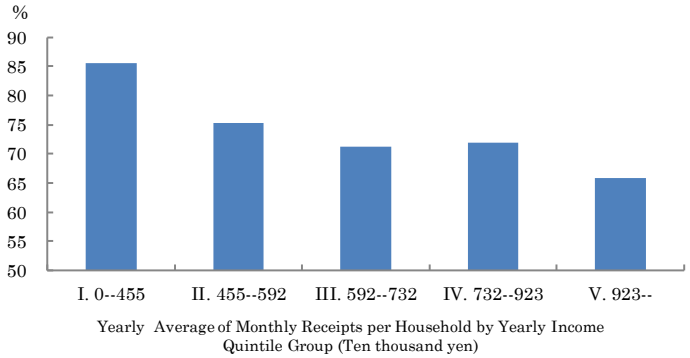

Figure 4. Average propensity to consume (workers' households)

Source: Family Income and Expenditure Survey 2017, Statics Bureau of Japan.
Notes: The average propensity consume is the ratio of consumption expenditures to disposable income.

For the sake of simplicity, we consider no government purchase and a closed economy in the sense that the trade of goods among countries is null. The equilibrium condition in the good market then satisfies

$$Y = N^s c^s + N^u c^u + I, \tag{5}$$

where I represents investment demand.

2.2. A benchmark: the case of full employment

The neoclassical growth models, such as the Solow model and the real business-cycle theory, suppose that commodity demand I expands automatically through price adjustment so that full employment holds.[5] As a benchmark, we begin with the full-employment case.

Assumption 1 (full employment):

$$l^s = l^u = 1, \qquad L^s = N^s, \qquad L^u = N^u.$$

This assumption rewrites (1) and (2) as

$$Y = f\left(\frac{\theta^s N^s}{\theta^u N^u}\right)\theta^u N^u, \tag{6}$$

$$w^s = \theta^s f'\left(\frac{\theta^s N^s}{\theta^u N^u}\right), \tag{7}$$

$$w^u = \theta^u\left[f\left(\frac{\theta^s N^s}{\theta^u N^u}\right) - f'\left(\frac{\theta^s N^s}{\theta^u N^u}\right)\frac{\theta^s N^s}{\theta^u N^u}\right]. \tag{8}$$

From (6), it is apparent that immigrants are helpful to push up aggregate income Y

[5] See Mankiw (2015, chapter 7) for the Solow growth model and King and Rebelo (1999) for a survey of the real business-cycle models.

regardless of whether they are skilled or not. Equations (7) and (8) implies that immigrant inflows also affect income distribution in the host country.[6]

Figures 5 graphically explain how immigrants change the wage rate in the labor market. The labor demand curves are given by (2), whereas the labor supply curve are now vertical at $L^i = N^i$. At the intersection E, the economy is in equilibrium satisfying either (7) or (8). As skilled labor supply N^s increases, the value of skilled labor w^s falls shifting the equilibrium to E' in figure 5-1. It also spurs the labor demand for the unskilled who complements the skilled in production, so that the value of unskilled labor w^u rises, as seen at E' in figure 5-2. Accepting more skilled workers from abroad is thus harmful to the skilled and beneficial to the unskilled in the host country. The same mechanism works for an inflow of the unskilled labor supply—i.e., an increase in N^u raises w^s and c^s but reduces w^u and c^u. (The formal proof is set out in appendix A.)

Proposition 1: *If the host country always attains full employment, immigrant inflows push up the domestic aggregate income but hurt some domestic workers.*

(a) Inflows of skilled workers are harmful to the domestic skilled and beneficial to the domestic unskilled.

(b) Inflows of unskilled workers are beneficial to the domestic skilled and harmful to the domestic unskilled.

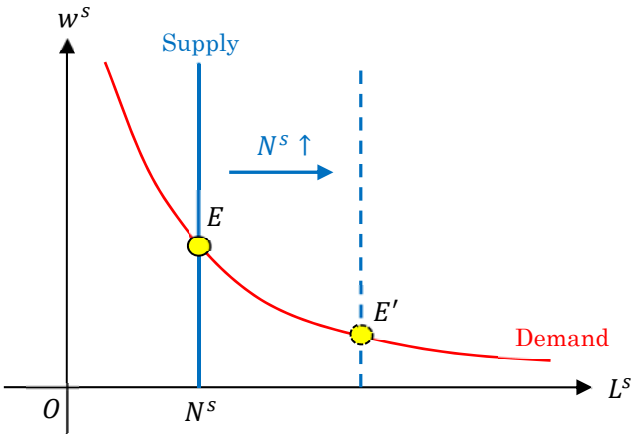

Figure 5-1. The skilled labor market.

[6] Equations (6) through (8) derive the investment level required to achieve full employment, I^f, as

$$I^f = (1-\sigma^s)\theta^s N^s f'\left(\frac{\theta^s N^s}{\theta^u N^u}\right) + (1-\sigma^u)\theta^u N^u \left[f\left(\frac{\theta^s N^s}{\theta^u N^u}\right) - f'\left(\frac{\theta^s N^s}{\theta^u N^u}\right)\frac{\theta^s N^s}{\theta^u N^u}\right].$$

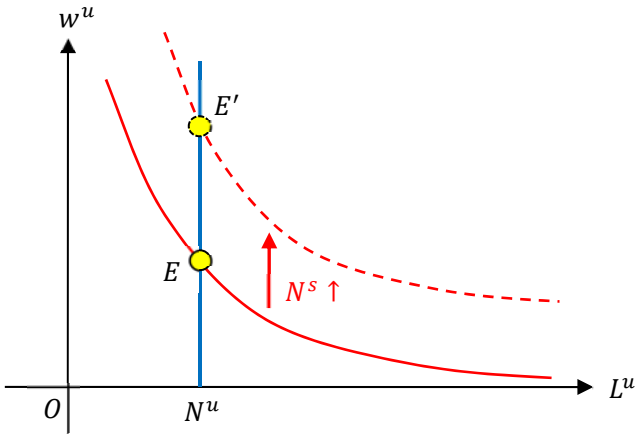

Figure 5-2. The unskilled labor market.

2.3. An effect of productivity improvement

An alternative way to offset population aging is to improve labor productivity. As indicated in figure 6, technological progress as well as capital accumulation contribute economic growth far beyond population growth.

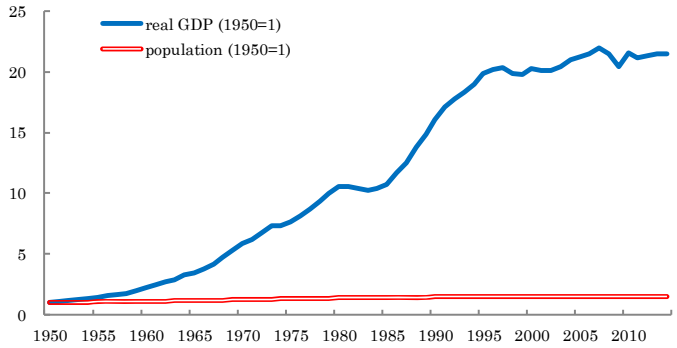

Figure 6-1. Real GDP and Population in Japan

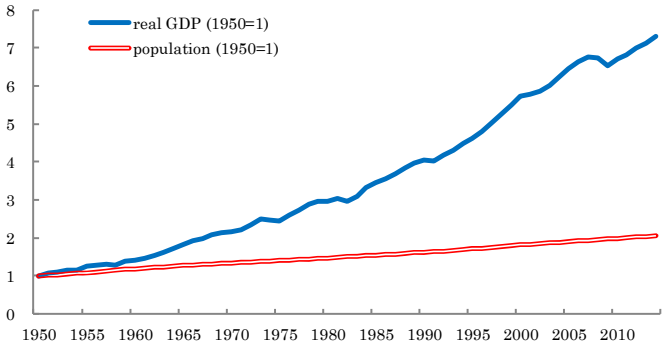

Figure 6-2. Real GDP and Population in United States of America

Source: Penn World Table, Version 9.0, Feenstra *et al.* (2015).
Notes: Real GDP and population are normalized to be unity for year 1950. Real GDP is the output-side one at chained PPPs (in millions 2011 US$).

While θ^s and N^s have qualitatively the same impacts on Y and w^u from (6) and (8), θ^s affects w^s in a different way through the following two channels. On the right-hand side in (7), a rise in θ^s improves the marginal productivity of skilled labor, holding $\theta^s N^s/\theta^u N^u$ fixed. It simultaneously diminishes the marginal product of skilled labor by increasing $\theta^s N^s/\theta^u N^u$. In general, the total effect is ambiguous depending on the elasticity of substitution between the two labor inputs, which is defined as

$$\epsilon \equiv -\frac{\mathrm{d}x}{x}\cdot\frac{w^s/w^u}{\mathrm{d}(w^s/w^u)} = \frac{f'(x)[f(x)-f'(x)x]}{-f(x)f''(x)x}\ \big(\in(0,\infty)\big),\ \text{where}\ \ x=\frac{\theta^s N^s}{\theta^u N^u}. \qquad (9)$$

If $\epsilon > (<)1$, skilled and unskilled labor are said gross substitutes (gross complements), which makes the labor demand curves flatter (steeper). The larger is ϵ, the less the marginal product of labor changes as $\theta^s N^s/\theta^u N^u$ changes. Empirical evidence suggests a relatively large value ($1 < \epsilon < \infty$), which implies that a rise in θ^i raises w^i $(i = s, u)$.[7]

The result is summarized in the following proposition:

Proposition 2: *If the host country always attains full employment and the elasticity of substitution between skilled and unskilled labor is plausibly large, the productivity improvement of both skilled and unskilled labor is Pareto-improving.*

(See appendix A for the mathematical proof.) It is noted that, in contrast with population growth, technological progress can enhance the welfare of all workers. Propositions 1 and 2 imply that accepting more immigrants should be implemented concurrently with job training to avoid the welfare losses of domestic workers.

3. The Keynesian Cross

3.1. The case of demand shortage

In *The General Theory of Employment, Interest and Money*, Keynes points out a possibility that inadequate commodity demand leads to stagnation with unemployment. In this section, we extend the benchmark model to the Keynesian cross, which is known as a simple tool to capture his insight.[8]

Suppose that investment demand per capita suddenly falls and remains unchanged at $\bar{\imath}$. Unemployment then occurs since nominal wages are rigid at $\bar{W}^s$ and $\bar{W}^u$.

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[7] For example, Ciccone and Peri (2005) find that ϵ takes around 1.5 using the U.S. state level data from 1950 to 1990. Ottaviano and Peri (2012) report that the elasticity between unskilled immigrants and domestic workers is between 6.5 and 20 in the U.S.

[8] See Mankiw (2015, chapter 10) for the Keynesian cross with a single consumer.

Assumption 2 (demand shortage and nominal wage rigidity):

$$I = (N^s + N^u)\bar{\iota}\ (>0), \qquad \bar{W}^s = Pw^s\ (>0), \qquad \bar{W}^u = Pw^u\ (>0),$$

where P *denotes the general price level and* $(\bar{\iota}, \bar{W}^s, \bar{W}^u)$ *are constant.*

Applying assumption 2 and eliminating P from (2) yields

$$\frac{\theta^s f'\left(\frac{\theta^s L^s}{\theta^u L^u}\right)}{\theta^u\left[f\left(\frac{\theta^s L^s}{\theta^u L^u}\right) - f'\left(\frac{\theta^s L^s}{\theta^u L^u}\right)\frac{\theta^s L^s}{\theta^u L^u}\right]} = \frac{\bar{W}^s}{\bar{W}^u},$$

implying that the fixed relative wages determine the labor demand ratio as

$$\frac{\theta^s L^s}{\theta^u L^u} \equiv x = x\left(\frac{\theta^s \bar{W}^u}{\theta^u \bar{W}^s}\right), \qquad x'(\cdot) > 0. \tag{10}$$

As $\bar{W}^u$ falls relative to $\bar{W}^s$, labor demand shifts from L^s to L^u. Given (10), the real wage rates satisfy

$$w^s = \theta^s f'(x), \qquad w^u = \theta^u[f(x) - f'(x)x]. \tag{11}$$

By substituting (10) into (1), we obtain the aggregate supply curve:

$$Y = f(x)\theta^u L^u.$$

Combining (2) through (5) and assumptions 2 gives the aggregate demand curve:

$$Y = g(x)\theta^u L^u + (N^s + N^u)\bar{\iota},$$

$$\text{where} \quad g(x) \equiv \sigma^s f'(x)x + \sigma^u[f(x) - f'(x)x],$$

$$0 < g(x) < f(x).$$

The two curves are described in figure 7-1. Aggregate income is in equilibrium at the intersection E, satisfying

$$Y^* = \frac{f(x)(N^s + N^u)\bar{\iota}}{f(x) - g(x)}. \tag{12}$$

Together with (3), (4), (10) and (11), it give

$$l^{s*} = \frac{x\bar{\iota}}{\theta^s[f(x) - g(x)]}\left(1 + \frac{N^u}{N^s}\right), \qquad l^{u*} = \frac{\bar{\iota}}{\theta^u[f(x) - g(x)]}\left(\frac{N^s}{N^u} + 1\right), \tag{13}$$

$$c^{s*} = \frac{\sigma^s f'(x)x\bar{\iota}}{f(x) - g(x)}\left(1 + \frac{N^u}{N^s}\right), \qquad c^{u*} = \frac{\sigma^u[f(x) - f'(x)x]\bar{\iota}}{f(x) - g(x)}\left(\frac{N^s}{N^u} + 1\right), \tag{14}$$

As $\bar{\iota}$ falls, aggregate income Y^* dampens, worsening individual employment l^{i*} and consumption c^{i*}. It supports the Keynes' view that commodity demand shortage is a cause of stagnation.

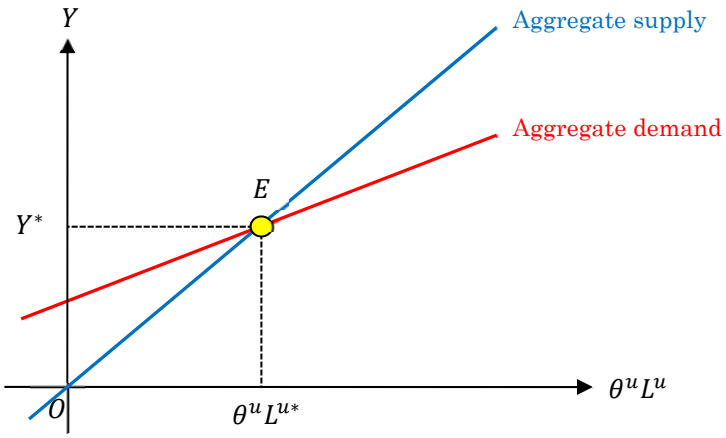

Figure 7-1. The Keynesian cross.

Regardless of whether immigrants are skilled or not, they push up aggregate investment demand and increase aggregate income Y^*. In the case of skilled labor inflows, employment and consumption of the unskilled expand, whereas the skilled is worse off because they have to share labor demand among more people. The opposite is also true—i.e., an increase in N^u raises l^{s*} and c^{s*} but reduces l^{u*} and c^{u*}.

Proposition 3: *If the host country faces demand shortage, immigrant inflows push up the domestic aggregate income but hurt some domestic workers.*

(a) Inflows of skilled workers are harmful to the domestic skilled and beneficial to the domestic unskilled.

(b) Inflows of unskilled workers are beneficial to the domestic skilled and harmful to the domestic unskilled.

(See appendix B for the proof.) Although propositions 1 and 3 look similar seemingly, the mechanism behind them fairly differs—the result in proposition 3 comes from not supply side but demand side of the economy. Obviously, an increase in $\bar{\iota}$ enriches all people expanding aggregate demand. To compensate the welfare losses from accepting immigration, the government concurrently has to implement the demand-stimulating policy.

3.2. An effect of labor reallocation

Let us now discuss how to stimulate aggregate demand by changing labor allocation. More concretely, we consider a reduction in the relative wage $\bar{W}^u/\bar{W}^s$, which shifts labor allocation from L^s to L^u from (10). The resulting decrease in $x(=\theta^s L^s/\theta^u L^u)$ alters the slopes of both aggregate supply and demand curves as follows.

(1) *Supply-side effect*: The decrease in x diminishes the productivity of unskilled labor, $f(x)$, making the aggregate supply curve flatter. Keeping the aggregate demand curve fixed, it increases Y and $\theta^u L^u$ (see figure 7-2).

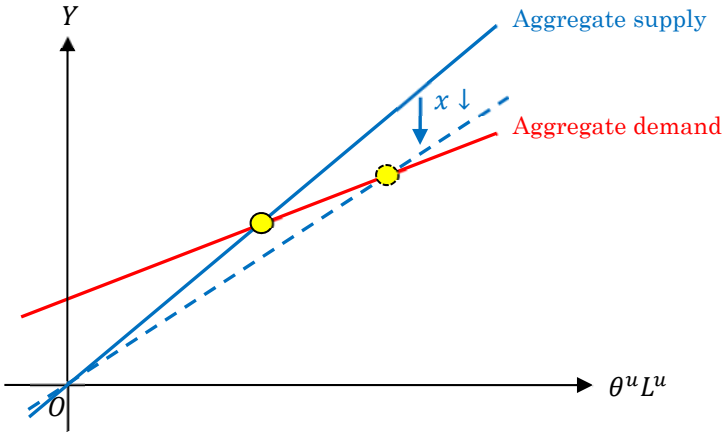

Figure 7-2. The supply-side effect of a decrease in x.

(2) *Demand-side effect*: As x decreases, w^s rises and w^u falls from (11). In addition, L^s declines for a given L^u from (10), so that $w^s L^s$ may or may not decrease. However, because the unskilled has a higher propensity to consume than the skilled, the aggregate demand definitely declines and becomes flatter. For a given aggregate supply curve, Y and $\theta^u L^u$ decrease (see figure 7-3).

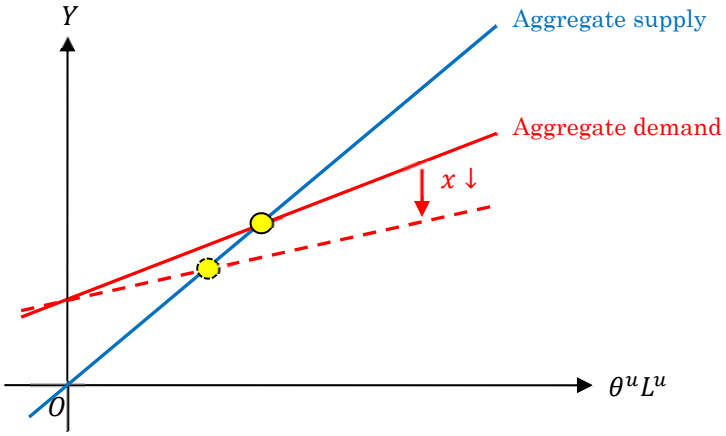

Figure 7-3. The demand-side effect of a decrease in x.

The total effect depends on the elasticity of substitution between skilled and unskilled labor, ϵ. With an empirically plausible value of ϵ, mentioned in section 2.3, the demand-side effect is so weak that the labor reallocation from L^s to L^u stimulates aggregate demand Y and unskilled consumption c^{u*} through the supply-side effect. We can also show that it decreases skilled consumption c^{s*} taking employment away from

them. (See appendix B.)

It is easy to confirm that a reduction in the relative productivity θ^s/θ^u similarly generates the labor reallocation from L^s to L^u, which enhances aggregate demand and unskilled consumption but deteriorates skilled consumption within $1 < \epsilon < \infty$. The results can be summarized as follows.

Proposition 4: *If the host country faces demand shortage and the elasticity of substitution between skilled and unskilled labor is plausibly large, the labor reallocation from the skilled to the unskilled push up aggregate demand but hurt some workers.*

(a) A cost reduction of hiring the unskilled relative to the skilled is harmful to the skilled and beneficial to the unskilled.

(b) The productivity improvement biased toward the unskilled is harmful to the skilled and beneficial to the unskilled.

(Appendix B provides the formal proof.)

Following propositions 3 and 4, it is noteworthy to mention some points. First of all, in the presence of unemployment, accepting immigrants must be carried out concurrently with the demand-stimulating policy so as not to hurt some domestic workers. It makes all people better off. If it is hard to implement the demand-side policy for some reasons, the government should set out the supply-side policy, which generates labor reallocation, as the next resort. For example, while inflows of unskilled workers hurt domestic unskilled worker, to provide employment to the unskilled—e.g., by the cost reduction of hiring them and the productivity improvement biased toward the unskilled—compensates their welfare losses.[9]

4. Conclusion

In this paper, we develop macroeconomic models with heterogenous labor to examine how immigrant inflows affect aggregate income and individual wage, employment and consumption in the host country. Immigrants are shown to hurt some domestic workers regardless of whether they are skilled or not, and independently of the presence or absence of unemployment. To avoid such an undesirable outcome, the government is required to take some sort of measures. If full employment is realized, it is productivity improvement, which makes all people better off.

[9] Note that the proportionate improvement of skill and unskilled labor productivity is neutral to aggregate income and individual consumption in the unemployment situation.

However, the employment situation should be considered when carrying out the policy. In the presence of unemployment due to demand shortage, the productivity improvement biased toward the skilled depresses aggregate demand since the skilled takes employment away from the unskilled who has a higher propensity to consume. It indicates the importance of employment creation under demand shortage rather than the supply-side policy. Increased employment of the unskilled, e.g., by the productivity improvement biased toward unskilled and a cost reduction of hiring the unskilled, expands aggregate demand. This result is particularly meaningful in the recent stagnant situations in developed countries such as Japan, European countries and the United States of America.

There are several directions of future research. First, we ignore the increasing cost of maintaining social security systems, which hampers capital accumulation (refer to Auerbach and Oreopoulos 1999 and Storesletten 2000 for this point). Second, more importantly, fertility decisions and technology progress are exogenous in the present model. However, immigration inflows may change incentives for bearing and raising children and investing resources in the R&D activity.[10] Finally, we have to analyze an open economy in which goods and financial assets are traded internationally. Population aging may affect the world interest rate and the relative prices of internationally traded commodities, which in turn change wages, employment and trade patterns of each country (see for example Domeij and Flodén 2006 and Naito and Zhao 2009).

[10] Futagami and Hori (2010) develops a model in which both technology progress and population growth are determined endogenously. Yakita (2017) presents various models with families' fertility decisions, which are affected by population aging and social security systems. Hashimoto and Ono (2011) analyzes the relationship of unemployment and pro-population policy in a dynamic context.

Appendix A: Proofs of Propositions 1 and 2

To simplify the notation, denote $x = \theta^s N^s / \theta^u N^u$. Totally differentiating (6) through (8) with respect to either N^s or N^u and keeping (3) in mind generates

$$\frac{\mathrm{d}Y}{\mathrm{d}N^s} = \theta^s f'(x) > 0, \qquad \frac{\mathrm{d}Y}{\mathrm{d}N^u} = \theta^u [f(x) - f'(x)x] > 0,$$

$$\frac{\mathrm{d}c^s}{\mathrm{d}N^s} = \sigma^s \frac{\mathrm{d}w^s}{\mathrm{d}N^s} = \frac{\sigma^s \theta^s f''(x)x}{N^s} < 0, \qquad \frac{\mathrm{d}c^s}{\mathrm{d}N^u} = \sigma^s \frac{\mathrm{d}w^s}{\mathrm{d}N^u} = -\frac{\sigma^s \theta^s f''(x)x}{N^u} > 0,$$

$$\frac{\mathrm{d}c^u}{\mathrm{d}N^s} = \sigma^u \frac{\mathrm{d}w^u}{\mathrm{d}N^s} = -\frac{\sigma^u \theta^u f''(x)x^2}{N^s} > 0, \qquad \frac{\mathrm{d}c^u}{\mathrm{d}N^u} = \sigma^u \frac{\mathrm{d}w^u}{\mathrm{d}N^u} = \frac{\sigma^u \theta^u f''(x)x^2}{N^u} < 0,$$

which proves proposition 1.

In the same way, we have the effects of productivity improvement:

$$\frac{\mathrm{d}Y}{\mathrm{d}\theta^s} = N^s f'(x) > 0, \qquad \frac{\mathrm{d}Y}{\mathrm{d}\theta^u} = N^u [f(x) - f'(x)x] > 0,$$

$$\frac{\mathrm{d}c^s}{\mathrm{d}\theta^s} = \sigma^s f'(x) \left[\frac{f'(x)x}{\epsilon f(x)} + \frac{\epsilon - 1}{\epsilon}\right], \qquad \frac{\mathrm{d}c^s}{\mathrm{d}\theta^u} = -\frac{\sigma^s \theta^s f''(x)x}{\theta^u} > 0,$$

$$\frac{\mathrm{d}c^u}{\mathrm{d}\theta^s} = -\frac{\sigma^u \theta^u f''(x)x^2}{\theta^s} > 0, \qquad \frac{\mathrm{d}c^u}{\mathrm{d}\theta^u} = \sigma^u [f(x) - f'(x)x] \left[\frac{f(x) - f'(x)x}{\epsilon f(x)} + \frac{\epsilon - 1}{\epsilon}\right],$$

where ϵ is given in (9). With $1 < \epsilon < \infty$, it holds that

$$\frac{\mathrm{d}c^s}{\mathrm{d}\theta^s} > 0, \qquad \frac{\mathrm{d}c^u}{\mathrm{d}\theta^u} > 0.$$

Thus, we obtain proposition 2.

Appendix B: Proofs of Propositions 3 and 4

Totally differentiating (12) through (14) with respect to either N^s or N^u proves proposition 3:

$$\frac{\mathrm{d}Y^*}{\mathrm{d}N^s} = \frac{f(x)\bar{\imath}}{f(x) - g(x)} > 0, \qquad \frac{\mathrm{d}Y^*}{\mathrm{d}N^u} = \frac{f(x)\bar{\imath}}{f(x) - g(x)} > 0,$$

$$\frac{\mathrm{d}c^{s*}}{\mathrm{d}N^s} = \sigma^s w^s \frac{\mathrm{d}l^{s*}}{\mathrm{d}N^s} = -\frac{\sigma^s f'(x)x\bar{\imath}N^u}{[f(x) - g(x)](N^s)^2} < 0, \qquad \frac{\mathrm{d}c^{s*}}{\mathrm{d}N^u} = \sigma^s w^s \frac{\mathrm{d}l^{s*}}{\mathrm{d}N^u} = \frac{\sigma^s f'(x)x\bar{\imath}}{[f(x) - g(x)]N^s} > 0,$$

$$\frac{\mathrm{d}c^{u*}}{\mathrm{d}N^s} = \sigma^s w^s \frac{\mathrm{d}l^{u*}}{\mathrm{d}N^s} = \frac{\sigma^u [f(x) - f'(x)x]\bar{\imath}}{[f(x) - g(x)]N^u} > 0, \qquad \frac{\mathrm{d}c^{u*}}{\mathrm{d}N^u} = \sigma^s w^s \frac{\mathrm{d}l^{u*}}{\mathrm{d}N^u}$$

$$= -\frac{\sigma^u [f(x) - f'(x)x]\bar{\imath}N^s}{[f(x) - g(x)](N^u)^2} < 0,$$

Using (9) through (11) and the definition of ϵ given in (9), we can prove

proposition 4:

$$\frac{\mathrm{d}x}{\mathrm{d}\bar{W}^s} = -x'\left(\frac{\theta^s\bar{W}^u}{\theta^u\bar{W}^s}\right)\frac{\theta^s\bar{W}^u}{\theta^u(\bar{W}^s)^2} < 0, \qquad \frac{\mathrm{d}x}{\mathrm{d}\bar{W}^u} = x'\left(\frac{\theta^s\bar{W}^u}{\theta^u\bar{W}^s}\right)\frac{\theta^s}{\theta^u\bar{W}^s} > 0,$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta^s} = x'\left(\frac{\theta^s\bar{W}^u}{\theta^u\bar{W}^s}\right)\frac{\bar{W}^u}{\theta^u\bar{W}^s} > 0, \qquad \frac{\mathrm{d}x}{\mathrm{d}\theta^u} = -x'\left(\frac{\theta^s\bar{W}^u}{\theta^u\bar{W}^s}\right)\frac{\theta^s\bar{W}^u}{(\theta^u)^2\bar{W}^s} < 0,$$

$$\frac{\partial Y^*}{\partial x} = \frac{(\epsilon-1)(\sigma^u-\sigma^s)f(x)f''(x)x(N^s+N^u)\bar{\iota}}{[f(x)-g(x)]^2} < 0 \quad \text{for} \quad 1<\epsilon<\infty,$$

$$\frac{\partial c^{s*}}{\partial x} = -\frac{(\epsilon-1)\sigma^s(1-\sigma^u)f(x)f''(x)x\bar{\iota}}{[f(x)-g(x)]^2}\left(1+\frac{N^u}{N^s}\right) > 0 \quad \text{for} \quad 1<\epsilon<\infty,$$

$$\frac{\partial c^{u*}}{\partial x} = \frac{(\epsilon-1)\sigma^u(1-\sigma^s)f(x)f''(x)x\bar{\iota}}{[f(x)-g(x)]^2}\left(\frac{N^s}{N^u}+1\right) < 0 \quad \text{for} \quad 1<\epsilon<\infty.$$

With $1<\epsilon<\infty$, it holds that

$$\frac{\mathrm{d}Y^*}{\mathrm{d}\bar{W}^s} > 0, \quad \frac{\mathrm{d}Y^*}{\mathrm{d}\bar{W}^u} < 0, \quad \frac{\mathrm{d}Y^*}{\mathrm{d}\theta^s} < 0, \quad \frac{\mathrm{d}Y^*}{\mathrm{d}\theta^u} > 0,$$

$$\frac{\mathrm{d}c^{s*}}{\mathrm{d}\bar{W}^s} < 0, \quad \frac{\mathrm{d}c^{s*}}{\mathrm{d}\bar{W}^u} > 0, \quad \frac{\mathrm{d}c^{s*}}{\mathrm{d}\theta^s} > 0, \quad \frac{\mathrm{d}c^{s*}}{\mathrm{d}\theta^u} < 0,$$

$$\frac{\mathrm{d}c^{u*}}{\mathrm{d}\bar{W}^s} > 0, \quad \frac{\mathrm{d}c^{u*}}{\mathrm{d}\bar{W}^u} < 0, \quad \frac{\mathrm{d}c^{u*}}{\mathrm{d}\theta^s} < 0, \quad \frac{\mathrm{d}c^{u*}}{\mathrm{d}\theta^u} > 0.$$

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