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Abstract

This paper provides a theoretical model to describe the prospects to end a civil war and to get out from the poverty trap in the region in which a rebel aims to appropriate a dictator's natural resource rents. The main protagonists are the dictator, the people, and the rebel. The dictator, who governs the formal sector of the economy, compels the people to contribute their labor to production and to defend the natural resource rents. The rebel is assumed to live in a remote area such as a mountain side, which is part of the informal sector.

One of our results indicates that if the productivity in the informal sector is sufficiently low or if the natural resource rents are sufficiently high, the rebel allocates all the labor to predation. In this case, the dictator may induce people to allocate some of their labor in order to defend the natural resource rents, thus causing a severe conflict, which can be considered as a poverty trap. An increase in the natural resource rents due to factors such as the discovery of a rare natural resource intensifies the conflict intensity in this case. We interpret the conflict intensity as a typical indicator of poverty trap because a civil war inevitably destructs an economy and people's life.

Our result suggests that foreign aid that improves the productivity in the informal sector or reinforcing the relative strength of the defense by the dictator's side may contribute to end a civil war. Some numerical examples are presented in the last section in order to illustrate the structure of our model.

Key words: civil war, natural resource rents, poverty trap, conflict intensity
JEL Classification Numbers: J29, P39

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1. Introduction

This paper provides a theoretical model to describe the prospects to end a civil war and to realize peace in the region in which a rebel aims to appropriate the dictator's natural resource rents.

The protagonists in this paper are a dictator who governs the formal economy, the people who works for the dictator, and a rebel who aims to appropriate the dictator's natural resource rents. The rebel is assumed to live in a remote area such as a mountain side, which is part of the informal sector. We assume that the dictator allows some foreign companies to mine diamonds, copper, or some other type of natural resource, which provides him with natural resource rents.

Olsson and Congdon Fors (2004) demonstrated that the institutional grievance of the formal and informal sectors, together with the relative strength of the ruler's defense, played a key role in the initiation of the Civil War in the Congo. They showed that an abundance of natural resources and the ruler's kleptocratic tendencies determined the conflict intensity. According to Olsson (2007), the conflict over diamond deposits is believed to have been a significant cause for the initiation, maintenance and prolongation of the Civil Wars in Angola, Sierra Leone, Liberia, and the Democratic Republic of the Congo.[1] It often happens that elites, or the governing class make use of the abundant natural resources to maintain their power. Bates (2008, p283) points out that elites from more prosperous regions can retain the loyalty of those from the less prosperous regions by channeling benefits to them, thus forestalling armed challenges. Collier and Gunning (2008, p213) points out that political leaders do not get the opportunity to loot because they are subject to various checks and balances, however, in many African countries, this does not hold true. Checks and balances do not have to be highly effective to stop outright looting. They stresses that the total failure of checks and balances in Africa occurred only in the context of military absolutism, such as with Mobutu, Abacah, and Amin, all coup leaders from the army.

On the basis of these ideas, we synthesize an economic theory of an institution, a theory of dictatorship, and a theory of conflict.

According to Aoki (2001, p10), an institution is a self-sustaining system of shared beliefs about a salient way in which the game is repeatedly played. Greif (2006, p30), stated that an institution is a system comprising certain social factors that conjointly generate regularity of behavior. Both Aoki and Greif note that the repetition of behaviors formed a pattern and they conceptualized this pattern as an institution. Since there have been several instances of struggle overt natural resource rents in the history of Africa,

[1] The Congo is rich in mineral resources. For the history and the situation of the Congo, see Leslie (1993) and Nest (2006). For natural resources and conflict in Sub-Saharan Africa, see Herbst (2000).

these struggles are rationalized to model the relationship between a dictator and a rebel as an institution.

Despite the importance of the subject of dictatorship, few studies in economics have shed light on it. Grossman (2002) proposed a model that indicated that if the technology of predation was sufficiently effective, the presence of a ruler, rather than the absence of one, would prove better for everyone concerned, including producers and predators. A king can enforce a collective choice concerning the allocation of resources in order to secure the producers' claims to their products. Grossman's paper explored the micro-foundation of strong power in a dictatorship. Acemoglu and Robinson (2000) proposed a model demonstrating that the enlargement of citizenship was a rational behavior of the powerful rich class in order to preempt revolution.

On the basis of the idea stated above, we present a model wherein a dictator compels the people to contribute their labor to production and defense.

This paper proceeds as follows. The model is presented in section 2, and the numerical examples of the model are presented in section 3. Section 4 summarizes the main results.

2. The Model

2.1 The Behavior of the Rebel

We assume an economy comprising a dictator, n numbers of homogenous people and m numbers of homogenous rebels. Since the people and the rebels are both homogenous, we can treat each of them as being one entity without loss of generality. The rebel, who probably lives in a remote area, belongs to the informal sector of the economy. The rebel is assumed to be endowed with l units of labor. Labor is allocated such that $l = L^R + L^P$, where L^P is the labor for production and L^R is the labor employed for predation. The production takes place under a linear production function. X^R denotes the rebel's output.

$$X^R = A^R(l - L^R)\ A^R > 0 \tag{1}$$

where A^R can be considered to reflect the production technology of the rebel. Since the rebel may be marginalized from the rest of the economy, his economy constitutes the informal sector. The rebel derives income from both production and predation, which is denoted by pD. In this expression, D denotes the total international market value of the rents obtained from natural resources such as diamonds and copper. We assume that some foreign companies mine diamonds or copper as well as pay natural resource rents D to the dictator. Out of this total value D, the rebel manages to appropriate a share of $0 \leq p \leq 1$. This share is represented by the following "predation success function" or "contest success function"[2].

$$p = \frac{L^R}{L^R + \theta L^D}, \theta > 0 \tag{2}$$

The variable L^D is the labor expended by the people in order to defend natural resource rents while θ reflects the relative strength of the defense. Using Equations (1) and (2), the rebel's income is denoted as follows.

$$Y^R = A^R(l - L^R) + \frac{L^R}{L^R + \theta L^D} D \tag{3}$$

The rebel maximizes his/her income by choosing to allocate the labor for predation. The first-order conditions for maximization are as follows.

$$-A^R + \frac{D\theta L^D}{(L^R + \theta L^D)^2} < 0 \quad L^R = 0 \tag{4}$$

$$-A^R + \frac{D\theta L^D}{(L^R + \theta L^D)^2} = 0 \quad 0 < L^R < l \tag{5}$$

$$-A^R + \frac{D\theta L^D}{(L^R + \theta L^D)^2} > 0 \quad L^R = l \tag{6}$$

Inequality (4) describes a corner solution, in which no predation occurs. Inequality (6) describes a corner solution, in which the rebel allocates all his labor to predation. Equation (5) describes an interior solution, in which the rebel allocates the labor to both production and predation. If we have a inner solution, $L^R > 0$, the first-order condition implies the following.

$$L^R = \sqrt{\frac{\theta D L^D}{A^R}} - \theta L^D \tag{7}$$

Equation (7) defines the rebel's reaction function to people's labor contribution to defense, that is, L^D. If it takes a positive value, the rebel allocates some labor to predation. Denoting the optimal level for the dictator of people's labor contribution to defense as L^D_*, we derive Lemma 1.

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[2] For the contest success function, see Hirshlifer (1991).

Lemma 1

The rebel allocates some of his labor to predation if $L_*^D < \frac{D}{\theta A^R}$.

If the labor devoted to defense, which induces the rebel to allocate some labor to predation exceeds the labor endowment of the people, the rebel always allocates labor to predation. In this case, the dictator cannot prohibit the rebel to predate the natural resource rents because labor endowment is insufficient. We derive Lemma 2 as follows.

Lemma 2

The rebel always allocates his labor to predation if $\frac{D}{\theta A^R} > \bar{L}$.

From Equation (7), we obtain Lemma 3.

Lemma 3

The rebel allocates all his labor to production if $L_*^D \geq \frac{D}{\theta A^R}$.

There may exist nine possible cases of equilibrium.

(1) Equilibrium with some conflict: The case where the rebel allocates some of his labor to predation and the dictator allocates part of people's labor to defense
(2) Equilibrium with severe conflict: The case where the rebel allocates all the labor to predation and the dictator allocates some of people's labor to defense
(3) Equilibrium in which the rebel gains control of the natural resource rents: The case where the rebel allocates all the labor to predation while the dictator renounces the defense of natural resource rents
(4) Equilibrium in which the dictator gains control of the natural resource rents: The case where the rebel allocates all the labor to production while the dictator allocates some of people's labor to defense
(5) Equilibrium with no conflict (1): The case where both the dictator and the rebel allocate all the labor to production
(6) Equilibrium with no conflict (2): The case where the rebel allocates all the labor to production while the dictator allocates all the people's labor to defense
(7) Equilibrium with severe conflict (2): The case where the rebel allocates some labor to predation, while the dictator allocates all the people's labor to defense
(8) Equilibrium with severe conflict (3): The case where the rebel allocates all the labor to predation while the dictator allocates all the people's labor to defense

(9) Equilibrium with some conflict (2): The case where the rebel allocates some labor to predation while the dictator allocates all the people's labor to production

(Insert Table 1 around here)

We analyze cases of (1), (2), (3), and (4) in the following. The equilibrium with severe conflict, or the high level of the conflict intensity explained below can be considered as a poverty trap. With respect to case (5), we consider this case as the one where the dictator renounces the natural resource rents at the outset and the rebel gains control without allocating the labor to predation. We can show that the level of the dictator's consumption in case (5) is lower than that of in case (3). Thus we conclude that the dictator does not choose this equilibrium. [3]

With respect to cases (6), (7), and (8), we show that the rebel does not allocate the labor to predation, if the dictator allocates some of people's labor to defense. There is no incentive for the dictator to allocate more amount of people's labor to defense than this level. Thus we conclude that the dictator does not choose cases (6), (7), and (8). We show that the dictator allocates some of people's labor to defense when the rebel allocates some labor to predation. Thus, we conclude that the dictator does not choose case (9). In the following, we analyze cases (1), (2), (3), and (4).[4]

First, we discuss the case of the equilibrium with some conflict.

2.2 Equilibrium with Some Conflict

Substituting Equation (7) into Equation (3), we obtain the following.

$$Y^R = A^R l + D - 2\sqrt{A^R \theta D L^D} + A^R \theta L^D \tag{8}$$

The structure of our game is as follows. In the beginning, the dictator informs the people about their share of production and the real wage with respect to their labor contribution to defense. Considering this, the people decide their labor allocation in order to maximize their income. Next, the dictator informs the rebel about people's labor contribution to defense. Considering this, the rebel then decides the amount of labor to be allocated to predation. Using backward induction to solve the game, we obtain a

[3] See Mathematical Appendix (1).

[4] See Table (1).

subgame perfect equilibrium.

The formal sector of the economy is governed by the dictator. Labor L is required to produce goods such as agricultural products. The production function is assumed to take the following Cobb- Douglas type. X denotes the output, and A denotes the productivity in the formal sector.

$$X = AL^{\gamma} \quad 0 < \gamma < 1, A > 0 \tag{9}$$

We assume that the people can contribute their labor to both production and the defense of the dictator's natural resource rents. The people are endowed with $\bar{L}$ units of labor. The labor devoted to defense is denoted by L^D, which is equal to $\bar{L} - L$. The dictator permits the people to obtain a certain amount of produced goods. Suppose that $\alpha(0 \leq \alpha \leq 1)$ is the share of production received by the dictator, and the remaining share, $1 - \alpha$, is the share of production received by the people. The dictator pays the real wage, denoted by w, for each unit of labor devoted to the defense of natural resource rents. The income of the people is defined as follows.

$$Y = (1 - \alpha)A(\bar{L} - L^D)^{\gamma} + wL^D \tag{10}$$

The people consume their entire income and decide their labor allocation in order to maximize their income. The first-order condition for optimality is as follows.

$$L^D = \bar{L} - \{\frac{\gamma(1 - \alpha)A}{w}\}^{\frac{1}{1-\gamma}} \tag{11}$$

We assume that the people's reservation income is given by the rebel's income, that is, Y^R. Using Equation (8), the people's participation constraint is obtained as follows.

$$(1 - \alpha)A(\bar{L} - L^D)^{\gamma} + wL^D \geq A^R l + D - 2\sqrt{A^R \theta D L^D} + A^R \theta L^D \tag{12}$$

The dictator can reduce the income of the people to the level of their reservation income level. Thus, inequality (12) is reduced to equality.

$$(1 - \alpha)A(\bar{L} - L^D)^{\gamma} + wL^D = A^R l + D - 2\sqrt{A^R \theta D L^D} + A^R \theta L^D \tag{13}$$

C denotes the dictator's consumption. The dictator's income comprises his share of production and the natural resource rents. The dictator's budget constraint can be denoted as follows.

$$C = \alpha A(\bar{L} - L^D)^\gamma + (1 - p)D - wL^D \tag{14}$$

Substituting Equations (2), (7), and (13) into Equation (14), we obtain the following.

$$C = A(\bar{L} - L^D)^\gamma + 3\sqrt{A^R\theta D L^D} - A^R\theta L^D - A^R l - D \tag{15}$$

The dictator derives utility from his consumption. For simplicity, we assume that the dictator's utility function takes the form of a logarithm.

$$U = \ln C \tag{16}$$

Substituting Equation (15) into Equation (16), we obtain the following.

$$U = \ln\{A(\bar{L} - L^D)^\gamma + 3\sqrt{A^R\theta D L^D} - A^R\theta L^D - A^R l - D\} \tag{17}$$

From the first-order condition for optimality, we obtain the following.[5]

$$-\gamma A(\bar{L} - L^D)^{\gamma-1} - A^R\theta + \frac{3}{2}\sqrt{\frac{A^R\theta D}{L^D}} = 0 \tag{18}$$

Equation (18) can be interpreted as follows. If the labor devoted to defense increases, the production decreases, whereas the wage payments and the share of natural resource rents increase. A decrease in the production and an increase in the wage payments can be interpreted as marginal cost, while an increase in the share of natural resource rents can be interpreted as marginal benefit. For the efficient allocation of labor, the marginal cost needs to be equated with marginal benefit.

Before performing comparative statics, we need to confirm the existence of an equilibrium. Using Equation (18), we define function F as follows.

$$F(L^D) = -\gamma A(\bar{L} - L^D)^{\gamma-1} - A^R\theta + \frac{3}{2}\sqrt{\frac{A^R\theta D}{L^D}} \tag{19}$$

Performing some algebraic calculations, we obtain the following.

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5 See Mathematical Appendix (2).

$$\frac{\partial F}{\partial L^D} = \gamma(\gamma-1)A(\bar{L}-L^D)^{\gamma-2} - \frac{3}{4}\sqrt{A^R\theta D}(L^D)^{\frac{-3}{2}} < 0 \tag{20}$$

$$F(0) = \infty \tag{21}$$

$$F(\bar{L}) = -\infty \tag{22}$$

Thus, we can depict Figure 1 as one of the possible examples of function F. L_*^D is the equilibrium value of labor devoted to defense that satisfies Equation (18).

(Insert Figure 1 around here)

If the equilibrium value of labor devoted to defense that satisfies Equation (18) takes a larger value than $\frac{D}{\theta A^R}$, the rebel allocates all the labor to production. Substituting $L^D = \frac{D}{\theta A^R}$ into Equation (19), we obtain the following.

$$F\left(\frac{D}{\theta A^R}\right) = -\gamma A(\bar{L} - \frac{D}{\theta A^R})^{\gamma-1} + \frac{A^R\theta}{2} \tag{23}$$

If Equation (23) takes a negative value, the equilibrium value of labor devoted to defense does not exceed the level at which the rebel stops predating. We can confirm the existence of the equilibrium value that satisfies Equation (18). Thus, we derive Lemmas 4 and 5.

Lemma 4

The equilibrium with some conflict does not exist if the following inequality does not hold.

$$\bar{L} < \frac{D}{\theta A^R} + (\frac{2\gamma A}{\theta A^R})^{\frac{1}{1-\gamma}} \tag{24}$$

Lemma 5

There exists an equilibrium in which the rebel allocates all the labor to production if the following inequality holds.

$$\bar{L} \geq \frac{D}{\theta A^R} + \left(\frac{2\gamma A}{\theta A^R}\right)^{\frac{1}{1-\gamma}}$$

(25)

Next, we perform comparative statics for the equilibrium. The results of the comparative statics are summarized in Table 2.

These results can be interpreted as follows. For example, consider the case of an increase in the natural resource rents. Other factors being the same, the marginal benefit increases. For the efficient allocation of labor, the marginal cost needs to increase. Thus, the labor devoted to production needs to be decreased.

Table 2

	D	$\bar{L}$	A	A^R	θ
L_*^D	$+$	$+$	$-$	$\pm$	$\pm$

According to Olsson and Congdon Fors (2004, p332), conflict intensity is defined as the total resources devoted to the struggle, that is, $L^R + L^{D*}$. In our model, this is shown as follows.

$$L^R + L_*^D = \sqrt{\frac{\theta D L_*^D}{A^R}} + (1-\theta)L_*^D$$

(26)

If $\theta \leq 1$, the equilibrium level of the conflict intensity decreases with A and increases with D and $\bar{L}$. From these results, we derive Proposition 1 and 2.

Proposition 1

In a dictatorship with a rebel and some conflict, the labor devoted to defense decreases with the productivity in the formal sector, and increases with the natural resource rents and the labor endowment in the formal sector.

Proposition 2

In a dictatorship with a rebel and some conflict, if $\theta \leq 1$, the conflict intensity decreases with the productivity in the formal sector and increases with the natural resource rents and the labor endowment in the formal sector.

As stated above, if $\theta \leq 1$, the conflict intensity increases with the natural resource rents. In many areas in which the civil war or a conflict over natural resource rents occurs, it is presumed that the relative strength of defense is not so significant, that is, $\theta \leq 1$. This might be one of the reasons that an increase in the natural resource rents induces a civil war because in that case, the rebel finds it rational to attack the dictator's natural resource rents.

In a situation where the relative strength of the defense is sufficiently low and the natural resource rents are high, we can predict that the conflict intensity will be high and this will lead to a poverty trap.

From Equation (11) and (13), we can obtain the equilibrium level of the share in production and the real wage for defense as follows.

$$\alpha^* = 1 - \frac{A^R l + D - 2\sqrt{A^R \theta D L^D_*} + A^R \theta L^D_*}{A(\bar{L} - L^D_*)^{\gamma-1}\{\bar{L} - (1-\gamma)L^D_*\}} \tag{27}$$

$$w^* = \frac{\gamma\{A^R l + D - 2\sqrt{A^R \theta D L^D_*} + A^R \theta L^D_*}{\bar{L} - (1-\gamma)L^D_*} \tag{28}$$

Next, we discuss the equilibrium with severe conflict.

2.3 Equilibrium with Severe Conflict

As stated above, the equilibrium with severe conflict occurs if inequality (6) holds. In this case, since the rebel allocates all the labor to predation, we obtain $L^R = l$. By performing some algebraic calculation, inequality (6) can be changed into the following.

$$A^R (L^R)^2 + 2A^R L^R \theta L^D + A^R \theta^2 (L^D)^2 - D\theta L^D < 0 \tag{29}$$

Substituting $L^R = l$ into (29), we obtain function $H(L^D)$ as follows.

$$H(L^D) = A^R \theta^2 (L^D)^2 + (2A^R l - D)\theta L^D + A^R l^2 \tag{30}$$

The solutions for $H(L^D) = 0$ are obtained as follows.

$$L^D_{H=0} = \frac{D - 2A^R l \pm \sqrt{D(D - 4A^R l)}}{2A^R \theta} \tag{31}$$

From Equation (31), if $D > 4A^R l$, the roots of Equation (30) are two real solutions that take positive values. If $D = 4A^R l$, the solution is $\frac{l}{\theta}$.

If $D < 4A^R l$, $H(L^D) > 0$. The roots of $H(L^D)$ are two complex conjugate numbers. Thus, Inequality (29) cannot hold and the rebel does not allocate all the labor to predation. We derive Figure 2 as follows.

(Insert Figure 2 around here)

If the labor devoted to defense exists in the following range, the rebel allocates all the labor to predation.

$$\frac{D - 2A^R l - \sqrt{D(D - 4A^R l)}}{2A^R \theta} < L_2^{D*} < \frac{D - 2A^R l + \sqrt{D(D - 4A^R l)}}{2A^R \theta} \tag{32}$$

Next, we analyze the behavior of the dictator. If the rebel allocates all the labor to predation, his/her income is given as the following.

$$Y^R = \frac{l}{l + \theta L^D} D \tag{33}$$

Using Equation (33), the participation constraint of the people is obtained as follows.

$$(1 - \alpha)A(\bar{L} - L^D)^\gamma + wL^D = \frac{lD}{l + \theta L^D} \tag{34}$$

Substituting Equation (34) into Equation (14), we obtain the following budget constraint of the dictator.

$$C = A(\bar{L} - L^D)^\gamma + \frac{\theta L^D - l}{l + \theta L^D} D \tag{35}$$

Substituting Equation (35) into Equation (16), and by performing some algebraic calculations, we obtain the following from the first-order conditions for optimization.

$$-\gamma A(\bar{L} - L^D)^{\gamma - 1} + \frac{2\theta l}{(l + \theta L^D)^2} D \leq 0 \;\; L_2^{D*} = 0$$

(36)

$$-\gamma A(\bar{L}-L^D)^{\gamma-1}+\frac{2\theta l}{(l+\theta L^D)^2}D=0 \quad 0<L_2^{D*}<\bar{L}$$

(37)

$$-\gamma A(\bar{L}-L^D)^{\gamma-1}+\frac{2\theta l}{(l+\theta L^D)^2}D\geq 0 \quad L_2^{D*}=\bar{L}$$

(38)

Inequality (36) describes a corner solution, wherein the dictator induces the people to contribute all their labor to production. In this case, we obtain $L_2^{D*}=0$. It appears that this case occurs when the productivity in the formal sector is sufficiently high, the labor endowment in the formal sector is sufficiently low, or the relative strength of the labor devoted to defense is sufficiently weak. In such circumstances, the dictator may find it unprofitable to defend natural resource rents, as compared to devoting all the labor to production. It is presumed that one of the ways to avoid this equilibrium is to reinforce the relative strength of the labor devoted to the defense.

Substituting $L^D=0$ into (36), we obtain the following.

$$\gamma A\bar{L}^{\gamma-1}\geq\frac{2\theta}{l}D$$

(39)

If inequality (39) is satisfied, the dictator renounces the natural resource rents. We obtain Lemma 6 as follows.

Lemma 6

Assume the case where the rebel allocates all the labor to predation. The dictator induces the people to allocate all their labor to production and renounces the natural resource rents if the following inequality holds.

$$\gamma A\bar{L}^{\gamma-1}\geq\frac{2\theta}{l}D$$

We analyze the case of an inner solution, in which Equation (37) holds. Before performing comparative statics, we need to confirm the existence of an equilibrium. Using Equation (37), we define function f as follows.

$$f(L^D)=-\gamma A(\bar{L}-L^D)^{\gamma-1}+\frac{2\theta l}{(l+\theta L^D)^2}D$$

(40)

By performing some algebraic calculations, we obtain the following properties of this function.

$$\frac{\partial f}{\partial L^D} = \gamma(\gamma - 1)A(\bar{L} - L^D)^{\gamma-2} - \frac{4\theta^2 lD}{(l + \theta L^D)^3} < 0$$

(41)

$$f(0) = -\gamma A\bar{L}^{\gamma-1} + \frac{2\theta D}{l}$$

(42)

$$f(\bar{L}) = -\infty$$

(43)

Function f is negatively sloped. If $f(0)$ takes a positive value, function f intersects with the horizontal line between $0 < L_2^{D*} < \bar{L}$. Assuming that $f(0)$ takes a positive value, we can confirm the existence of an equilibrium. Figure 3 is one of the possible examples of function f.

(Insert Figure 3 around here)

Figure 3 indicates that if $f(0)$ takes a non-positive value, the equilibrium level of the labor devoted to defense cannot take a positive value. This condition is given by Lemma 6. In addition to this, we should note that Equation (32) needs to be satisfied. Substituting (31) into function f, we obtain the followings.

$$-\gamma A\{\bar{L} - \frac{D - 2A^R l - \sqrt{D(D - 4A^R l)}}{2A^R\theta}\}^{\gamma-1} + \frac{2\theta lD}{\{l + \frac{D - 2A^R l - \sqrt{D(D - 4A^R l)}}{2A^R}\}^2} > 0$$

(44)

$$-\gamma A\{\bar{L} - \frac{D - 2A^R l + \sqrt{D(D - 4A^R l)}}{2A^R\theta}\}^{\gamma-1} + \frac{2\theta lD}{\{l + \frac{D - 2A^R l + \sqrt{D(D - 4A^R l)}}{2A^R}\}^2} < 0$$

(45)

If inequalities (44) and (45) are not satisfied, inequality (6) cannot hold, and the rebel will not allocate all his labor to predation. We obtain Lemma 7 as follows.

Lemma 7

Assume the case where the rebel allocates all the labor to predation. The dictator induces the people to allocate their labor to both production and defense of the natural resource rents if the following inequalities hold.

$$-\gamma A\bar{L}^{\gamma-1}+\frac{2\theta D}{l}>0$$

$$-\gamma A\{\bar{L}-\frac{D-2A^{R}l-\sqrt{D(D-4A^{R}l)}}{2A^{R}\theta}\}^{\gamma-1}+\frac{2\theta lD}{\{l+\frac{D-2A^{R}l-\sqrt{D(D-4A^{R}l)}}{2A^{R}}\}^{2}}>0$$

$$-\gamma A\{\bar{L}-\frac{D-2A^{R}l+\sqrt{D(D-4A^{R}l)}}{2A^{R}\theta}\}^{\gamma-1}+\frac{2\theta lD}{\{l+\frac{D-2A^{R}l+\sqrt{D(D-4A^{R}l)}}{2A^{R}}\}^{2}}<0$$

In this case the equilibrium level of the labor devoted to defense is obtained from Equation (37). Next, we perform comparative statics by using the main exogenous variables. The results are summarized in Table 4.

Table 3

	D	$\bar{L}$	A	l	θ
L_2^{D*}	+	+	−	±	±

From Table 3, we obtain Proposition 3.

Proposition 3

In a dictatorship with a rebel and severe conflict, the labor devoted to defense decreases with productivity in the formal sector and increases with the natural resource rents and the labor endowment in the formal sector.

In this case, the conflict intensity is denoted by $l+L_2^{D*}$. From these results, we derive Proposition 4.

Proposition 4

In a dictatorship with a rebel and severe conflict, the conflict intensity increases with the natural resource rents, and the labor endowment in the formal sector, whereas it decreases with the productivity in the formal sector.

This situation may correspond to the Civil War in the West Africa. Since the rebel's productivity was sufficiently low and the natural resource rents were sufficiently high, the rebel allocated all the labor to predation. Since the dictator had enough power to defend the natural resource rents, a severe conflict might have occurred in West Africa. In such a situation, an increase in the natural resource rents, which is the result of factors such as new discoveries, increases the conflict intensity. This was the reason that the people in that region were trapped in the poverty trap.

With respect to the equilibrium level of the share in production and the real wage for defense, using Equations (11) and (37), we obtain the following.

$$\alpha^* = 1 - \frac{lD(\bar{L} - L_2^{D*})^{1-\gamma}}{(l + \theta L_2^{D*})A\{\bar{L} - (1-\gamma)L_2^{D*}\}} \tag{46}$$

$$w^* = \frac{lD\gamma}{(l + \theta L_2^{D*})\{\bar{L} - (1-\gamma)L_2^{D*}\}} \tag{47}$$

Next, we analyze the equilibrium in which the rebel gains control of the natural resources.

2.4 Equilibrium Where the Rebel Gains Control of Natural Resource: The Case Where the Rebel Allocates all the Labor to Predation While the Dictator Renounces the Defense of Natural Resource Rents

This equilibrium occurs when inequality (6) holds. We need to note the following. In this equilibrium, the dictator allocates all the people's labor to production. If there is no labor for defense, the rebel does not allocate labor to predation. It appears that the rebel can obtain all the natural resource rents without allocating labor to predation. However, we have already shown that the dictator does not choose the equilibrium in which both the dictator and the rebel allocate all the labor to production in Mathematical Appendix (1). We suppose that the dictator has an advantage in some types of negotiations with the rebel. The dictator can propose that the rebel be given all the natural resource rents as long as he allocates all the labor to predation. If the natural resource rents are more than the income in the equilibrium with severe conflict, the rebel accepts the dictator's offer.

In this equilibrium, the income of the rebel is given by natural resource rents D. The reservation income of the people is also given by D. The participation constraint of the people is given as follows.

$$(1-\alpha)A\bar{L}^{\gamma} = D \tag{48}$$

The share of production received by the people is given as follows.

$$\alpha = \frac{A\bar{L}^{\gamma} - D}{A\bar{L}^{\gamma}}$$

(49)

The dictator informs the people about the zero level of real wage for defense. The dictator's consumption is obtained as follows.

$$C = A\bar{L}^{\gamma} - D$$

(50)

Equation (37) describes an inner solution, wherein the dictator induces the people to allocate some of their labor to defense. We must note that it is not profitable for the dictator to induce the people to allocate all their labor to defense since the first term of Equation (38) takes a negative infinite value when $\bar{L} = L_3^{D*}$ Thus, inequality (36) cannot be satisfied.

In this equilibrium the conflict intensity is given by l. It is possible that this equilibrium is one of the good equilibriums since its conflict intensity may be low compared to that in the other equilibrium.

Next, we analyze the case where the rebel allocates all the labor to production, that is, the equilibrium in which the dictator gains control of the natural resources.

2.5 Equilibrium Where the Dictator Gains Control of the Natural Resource: The Case Where the Rebel Allocates All the Labor to Production While the Dictator allocates Some of People's Labor to Defense

As stated above, this equilibrium occurs when inequality (4) holds. For example, we can interpret that this is the situation in which the rebel has a sufficiently high level of productivity. In this case, the rebel allocates all the labor to production. The rebel's income is denoted as follows.

$$Y^R = A^R l$$

(51)

The people's participation constraint, Equation (13), changes into the following.

$$(1-\alpha)A(\bar{L} - L^D)^{\gamma} + wL^D = A^R l$$

(52)

The dictator receives all the natural resource rents. Thus, we obtain $p = 0$. Using this and Equations (14) and (52), the dictator's budget constraint can be obtained as follows.

$$C = A(\bar{L} - L^D)^{\gamma} + D - A^R l$$

(53)

Substituting Equation (53) into the dictator's utility function, we obtain Equation (54).

$$U = \ln\{A(\bar{L} - L^D)^\gamma + D - A^R l\}$$

(54)

From the first-order condition for optimization, we obtain the following.

$$-\frac{\gamma A(\bar{L} - L^D)^{\gamma-1}}{C} < 0$$

(55)

Thus, we obtain the solution of no conflict, that is, $L_4^{D*} = \frac{D}{\theta A^R}$. This is the minimum level of labor devoted to defense that makes the rebel allocate all the labor to production, as is explained in Lemma 3. In this case, the conflict intensity is obtained as $L_4^{D*} = \frac{D}{\theta A^R}$. It is possible that this equilibrium is one of good equilibriums since its conflict intensity may be low compared to that is the other equilibrium.

In this case, the dictator sets the real wage at the following level, inducing the people to allocate all their labor to production. The dictator sets the share in production, where the following holds.

$$w^* = \frac{\gamma A^R l}{\bar{L} - (1-\gamma)\frac{D}{\theta A^R}}$$

(56)

$$\alpha^* = 1 - \frac{A^R l}{A(\bar{L} - \frac{D}{\theta A^R})^{\gamma-1}\{\bar{L} - (1-\gamma)\frac{D}{\theta A^R}\}}$$

(57)

The share in production needs to take a positive value. Next, we present the numerical examples of the model.

3. Numerical Examples of the Model

In this section, we present the numerical examples of the model in order to clearly illustrate the structure of our model. First, we assume that the parameters of the model take the following values.

$$\theta = 1 \quad \gamma = 0.4 \quad l = 4 \quad D = 4 \quad A^R = 1.5 \quad \bar{L} = 10 \quad A = 8$$

By substituting the above values, we compute the dictator's consumption level in each case as follows.

(1) Equilibrium with some conflict: $C = 15.794494$
(2) Equilibrium with severe conflict: $C = 17.047066$
(3) Equilibrium in which the rebel gains control of the natural resource rents: $C = 16.0959$
(4) Equilibrium in which the dictator gains control of the natural resource rents: $C = 15.7504986$

3.1 Equilibrium with Some Conflict

We need to note that both the equilibrium with severe conflict and that in which the rebel gains control of the natural resources rents cannot exist under the preceding values of the parameter. We derive $D < 4A^R l$ so that Equation $H(L^D)$ has imaginary solutions. Thus, the equilibrium of $L_*^R = l$ is not chosen by the rebel, and the equilibria of (2) and (3) cannot be realized. The dictator chooses the equilibrium with some conflict since he can enjoy a higher level of consumption than that in the equilibrium in which the dictator gains control of the natural resource rents. The dictator informs the people about the following level of the share in production and the real wage for defense in order to induce the equilibrium with some conflict.

$$\alpha = 0.70620083 \quad w = 0.275630054$$

Considering this, the people choose the following level of labor for defense.

$$L_*^D = 2.271$$

This equilibrium level of labor for defense can be approximately computed from Equation (18) by substituting the above values of the parameters. Considering this value of labor for defense, the rebel chooses the following level of predation.

$$L_*^R = 0.189894$$

The rebel's income is obtained as $Y^R = 6.023818$ by substituting the above values into Equation (8).The dictator's consumption is obtained as $C = 15.794494$.The conflict intensity in this case is obtained as follows.

$$L_*^D + L_*^R = 2.460894$$

In these values of the parameter, the dictator finds it profitable to allow the rebel to predate the natural resource rents. We show that the equilibrium with some conflict exists in other values of the parameters. Suppose that the natural resource rents take a high value, and the productivity in the informal sector takes a higher value. Assume that the parameter of the model takes the following values.

$$\theta = 1 \;\; \gamma = 0.4 \;\; l = 4 \;\; D = 27 \;\; A^R = 2 \;\; \bar{L} = 10 \;\; A = 8$$

The dictator informs the people about the following level of their share in production and the real wage for defense, in order to induce the equilibrium with some conflict.

$$w = 0.71666953 \quad \alpha = 0.635683811$$

In view of this, the people choose the following level of labor for defense.

$$L^D_* = 7.75$$

This equilibrium level of labor for defense can be approximately computed from Equation (18) by substituting the above values of the parameter. Considering this value of labor for defense, the rebel chooses the following level of predation.

$$L^R_* = 2.478636$$

The rebel's income is obtained as $Y^R = 9.584549$ by substituting the above values into Equation (8). The dictator's consumption is obtained as $C = 21.937113$. Conflict intensity in this case is obtained as follows.

$$L^D_* + L^R_* = 10.228636$$

We find that an increase in the natural resource rents increases the conflict intensity with respect to these values of parameters. Thus, what will happen if the productivity in both the areas improves?

We show that the equilibrium with some conflict exists in the other values of the parameters. Suppose that the productivity in both the areas takes higher values and that the parameters of the model take the following values.

$$\theta = 1 \;\; \gamma = 0.4 \;\; l = 4 \;\; D = 27 \;\; A^R = 13 \;\; \bar{L} = 10 \;\; A = 60$$

The dictator informs the people about the following level of their share in production and the real wage for defense, in order to induce the equilibrium with some conflict.

$$\alpha = 0.656977566 \quad w = 2.363466868$$

Considering this, the people choose the following level of labor for defense.

$$L^D_* = 1.996$$

This equilibrium level of labor for defense can be approximately computed from Equation (18) by substituting the above values of parameters. Considering this value of labor for defense, the rebel chooses the following level of predation.

$$L^R_* = 0.04006$$

The rebel's income is obtained as $Y^R = 52.0145$. The dictator's consumption is obtained as $C = 112.32892$. The conflict intensity in this case is obtained as follows.

$$L_*^D + L_*^R = 2.03606$$

3.2 Equilibrium with Severe Conflict

We show the equilibrium with severe conflict by substituting the other appropriate values of the parameters. Assume that the parameters of the model take the following values.

$$\theta = 1 \;\; \gamma = 0.4 \;\; l = 4 \;\; D = 27 \;\; A^R = 1.5 \;\; \bar{L} = 10 \;\; A = 8$$

In comparison with the first case of the equilibrium with some conflict, the value of natural resource rents increases. In this case, we obtain $D > 4A^R l$. Equation $H(L^D)$ has real solutions. Thus, the equilibrium of $L_*^R = l$ can be chosen by the rebel. In addition to this, we can confirm that the following does not hold with respect to the above values of parameters.

$$\gamma A \bar{L}^{\gamma-1} \geq \frac{2\theta D}{l}$$

Thus, we confirm that the equilibrium in which the rebel gains control of the natural resource rents do not hold under these circumstances. We also confirm that the following inequality holds in the above values of parameters.

$$\frac{D}{\theta A^R} > \bar{L}$$

This shows that Lemma 2 holds. Thus the equilibrium in which the dictator gains control of the natural resource rents does not hold, either. We may obtain the equilibrium with severe conflict. The dictator informs the people about the following level of the share in production and the real wage for defense, in order to induce the equilibrium with severe conflict.

$$\alpha = 0.60639353 \;\; w = 0.084884306$$

Considering this, the people choose the level of labor for defense, $L_2^{D*} = 7.2$.This equilibrium level of labor for defense can be approximately computed from Equation (39) by substituting the above values of the parameters. Considering this value of labor for defense, the rebel allocates all the labor to predation, $L_*^R = l = 4$.The rebel's income is obtained as $Y^R = 9.6428571$. The dictator's consumption is obtained as $C = 19.791123$.

Substituting the following values, we can confirm that inequality (6) holds.

$$L_2^{D*} = 7.2 \;\; L_*^R = l = 4 \;\; A^R = 1.5 \;\; D = 27 \;\; \theta = 1$$

The conflict intensity in this case is obtained as follows.

$$L_2^{D*} + l = 11.2$$

3.3 Equilibrium in Which the Rebel Gains Control of the Natural Resource Rents

In this case, as stated in 2.4, the dictator allows the rebel to obtain all the natural resource rents as long as the rebel allocates all the labor to predation. We show the equilibrium in which the rebel gains control of natural resource rents by substituting the appropriate values of the parameters. Assume that the parameters of the model take the following values.

$$\gamma = 0.4 \;\; A_R = 1.5 \;\; l = 4 \;\; A = 134.35 \;\; \bar{L} = 10 \;\; D = 27 \;\; \theta = 1$$

First, we compute the optimal level of the share in production and the real wage by way of substituting the above values using Equations in the equilibrium with severe conflict. The dictator informs the people about the following level of their share in production and the real wage for defense, in order to induce the equilibrium with severe conflict.

$$\alpha = 0.919996349 \;\; w = 0.00803846$$

In view of this, the people choose the level of labor for defense, $L_3^{D*} = 0.00015$. This equilibrium level of labor for defense can be approximately computed from Equation (39). Considering this value of labor for defense, the rebel allocates all the labor to predation, $L_*^R = l = 4$. The rebel's income is obtained as $Y^R = 26.9989875$. The dictator's consumption is obtained as $C = 310.47194$.

The conflict intensity in this case is obtained as follows.

$$L_3^{D*} + L_*^R = 4.00015$$

These values of equilibrium are almost the same as those of the equilibrium in which the rebel gains control of the natural resource rents, that is, $L_3^{D*} = 0$. It is evident that the rebel's income is obtained as $Y^R = 27$. The dictator informs the people about the following level of the share in production and the real wage for defense, in order to induce the equilibrium in which the rebel gains control of the natural resource rents.

$$\alpha = 0.91993 \;\; w = 0$$

Using Equation (53), the dictator's consumption is obtained as follows.

$$C = 310.4719$$

The conflict intensity in this case is obtained as follows.

$$L_3^{D*} + L_*^R = l = 4$$

3.4 Equilibrium Where the Dictator Gains Control of the Natural Resource Rents

This equilibrium is obtained when inequality (4) holds, for example, the case where the relative strength of defense takes a sufficiently high value. Assume that the parameters of the model take the following values.

$$\theta = 1.24 \ \ \gamma = 0.4 \ \ l = 4 \ \ D = 4 \ \ A^R = 1.5 \ \ \bar{L} = 10 \ \ A = 8$$

If the relative strength of defense θ takes a value more than 1.24 without changing the value of the other parameters compared with the equilibrium of some conflict, inequality (4) is realized and no predation occurs, $L_*^R = 0$. The equilibrium in which the dictator gains control of the natural resource rents is realized. In this equilibrium of the preceding values of parameters ($\theta = 1.24$), the rebel's income is obtained as $Y^R = A^R l = 6$. The dictator allocates labor for defense to the minimum level, which makes the rebel renounce predation, that is, $L_4^{D*} = \frac{D}{\theta A^R} = 2.150538$. The dictator's consumption is obtained as $C = 16.2400469$ from Equation (57). The dictator informs the people about the following level of their share in production and the real wage for defense, in order to induce the equilibrium in which the dictator gains control of natural the resource rents, $L_4^{D*} = \frac{D}{\theta A^R} = 2.150538$.

$$w = 0.70354202 \ \ \alpha = 0.27555556$$

The conflict intensity in this case is obtained as follows.

$$L_4^{D*} + L_*^R = 2.150538$$

4. Concluding Remarks

This paper provided a theoretical model to describe the prospects to end a civil war and to realize peace in the region in which a rebel aimed to appropriate the dictator's natural resource rents. We obtained the following four cases of equilibrium.

(1) Equilibrium with some conflict: The case where the rebel allocates some of his labor

to predation
(2) Equilibrium with severe conflict: The case where the rebel allocates all the labor to predation and the dictator induces the people to allocate their labor to both production and defense for natural resource rents.
(3) Equilibrium where the rebel gains control of natural resource rents: The case where the rebel allocates all the labor to predation and the dictator induces the people to allocate all their labor to production.
(4) Equilibrium where the dictator gains control of the natural resource rents: The case where the rebel allocates all the labor to production

It is evident that the dictator chooses an equilibrium that provides the highest level of utility. One of our results shows that if the productivity in the informal sector is sufficiently low or natural resource rents are sufficiently high, the rebel allocates all the labor to predation. In this case, the dictator may induce the people to allocate some of their labor to the defense of natural resource rents. This causes a severe conflict, so to say, the poverty trap. In such a situation, an increase in the natural resource rents due to factors such as the discovery of a rare natural resource such as a rare metal, intensifies the conflict intensity. This result may explain the situation of the Civil War in the West Africa. A severe conflict or high level of conflict intensity can be considered as a poverty trap because a civil war inevitably makes the people poor.

In this situation, if foreign countries wish to end a severe conflict or a civil war in a particular area, they should take certain measures to improve the productivity in that area. If foreign countries considerably help the rebel to ameliorate productivity in his/her governing sector, it is presumed that the rebel will stop predating, and will allocate all the labor to production. According to our results, this leads to the equilibrium with no conflict.

If foreign countries help the dictator to ameliorate productivity in the formal sector, it is presumed that the labor devoted to defense will decrease. This can weaken the conflict intensity. However, it does not necessarily imply that this will lead to the equilibrium with no conflict. The other way to avoid the equilibrium with severe conflict may be to reinforce the relative strength of defense.

It should be noted that our conclusion depends on the assumption that predation and defense do not involve the loss of human life. In reality, both predation and defense entail considerable sacrifice. If this fact is taken into consideration, our conclusion may change.

Mathematical Appendix

(1) In the equilibrium with no conflict (1), the dictator renounces natural resource rents

and the rebel gains control of the same without allocating the labor to predation. In this case, the income of the rebel is obtained as follows.

$$Y^R = A^R l + D$$

The income of the people is obtained as $(1-\alpha)A\bar{L}^\gamma$. The participation constraint of people is obtained as the following.

$$(1-\alpha)A\bar{L}^\gamma = A^R l + D$$

The dictator's consumption is obtained as follows.

$$C = A\bar{L}^\gamma - A^R l - D$$

This is smaller than the consumption level shown by Equation (50) in the case where the rebel gains control of the natural resource rents. Thus, there is no incentive for the dictator to induce the equilibrium with no conflict (1).

(2)

The first-order condition for optimality is as follows.

$$\frac{\partial U}{\partial w} = \{-\gamma A(\bar{L} - L^D)^{\gamma-1} - A^R\theta + \frac{3}{2}\sqrt{\frac{A^R\theta D}{L^D}}\}\left(\frac{\partial L}{\partial w}\right) = 0$$

Abbreviating $\frac{\partial L}{\partial w}$, we obtain Equation (18).

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Table 1

Equilibrium with some conflict

	Production	Predation or Defense
The rebel	○	○
The dictator	○	○

Equilibrium with severe conflict

	Production	Predation or Defense
The rebel		○
The dictator	○	○

Equilibrium with no conflict (1)

	Production	Predation or Defense
The rebel		○
The dictator	○	

Equilibrium with no conflict (2)

	Production	Predation or Defense
The rebel	○	
The dictator	○	○

Figure 1

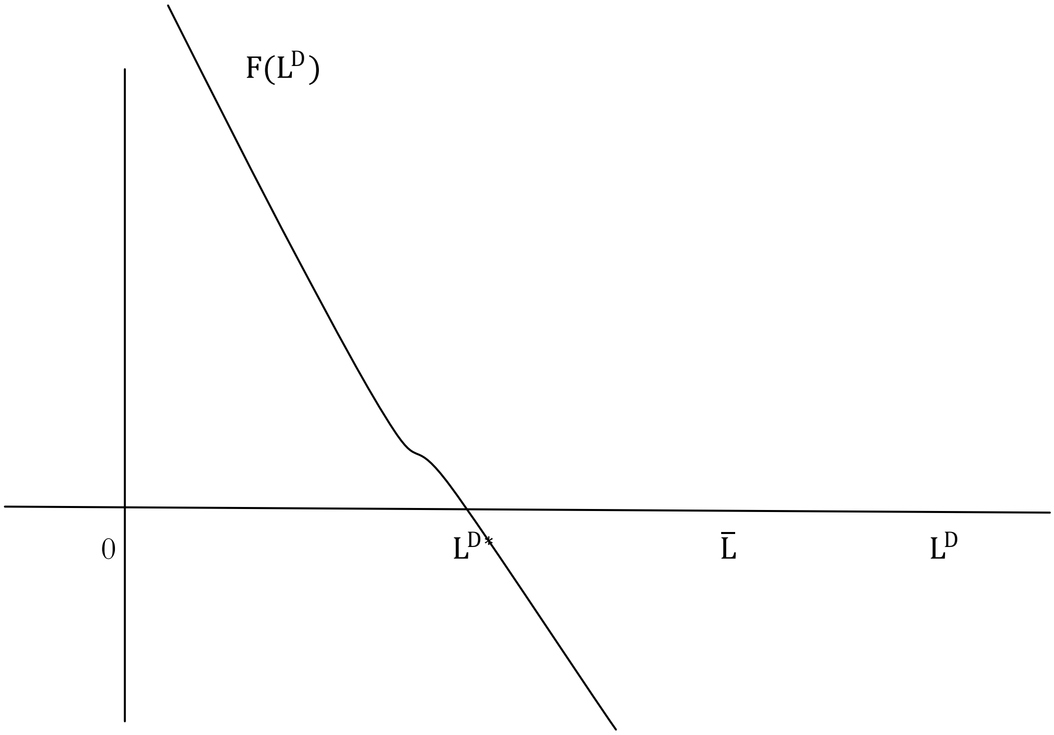

Figure 2

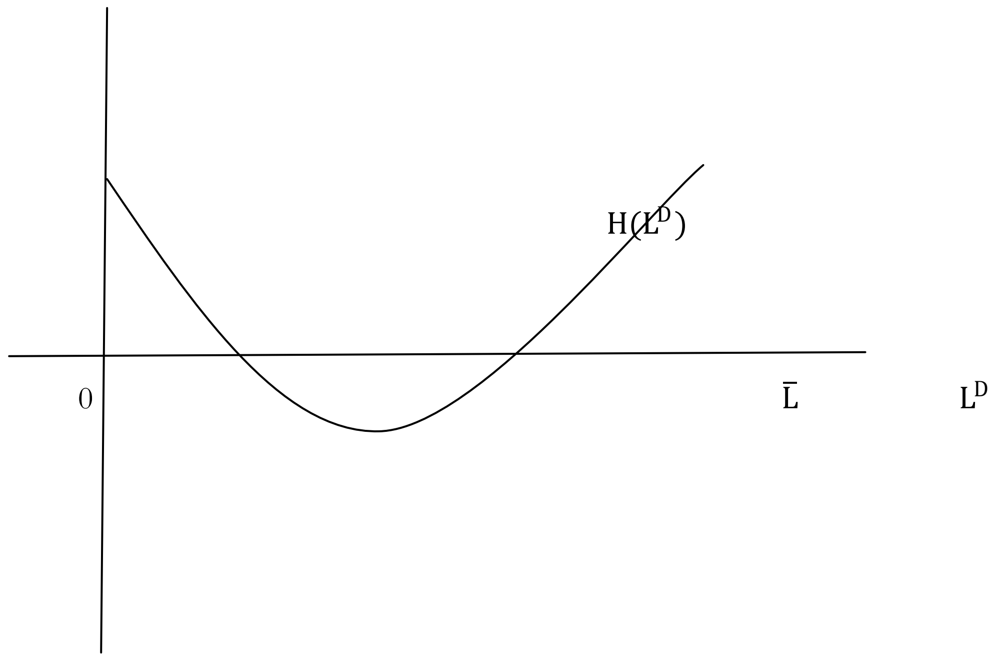

Figure 3