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Abstract

This paper examines whether an efficient outcome can be achieved through the bilateral contracting processes in a noncooperative coalitional bargaining game model with externalities and renegotiations. We describe the bargaining situation in a strategic form game. When the members of coalitions make binding agreements about their actions and transfers in the coalition formation process, almost all Markov perfect equilibria converge to the efficient state. On the other hand, in the partition function form game situation, all equilibria may remain in an inefficient state forever even if the grand coalition is efficient.

Key words: Coalitional bargaining, bilateral contracting, externalities, strategic form game

JEL Classification: C72, C78, D62

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1 Introduction

This paper considers whether a Pareto efficient allocation in economies with externalities is ultimately realized through dynamic processes of bargaining over contracts. The Coase theorem states that an efficient outcome is achieved through voluntary negotiations among agents when bargaining frictions (arising, for example, from private information, bargaining costs and imperfect recall) are insignificant. Recently, the Coase theorem has been discussed in the framework of noncooperative bargaining game models with externalities and renegotiations. Gomes (2005) showed that if the grand coalition is efficient, no delay of agreements occurs and the economy converges to the grand coalition through multilateral contracting processes. Although Gomes only allows expansions of coalitions for the negotiations, Gomes and Jeheil (2005) considered the general process of coalition formation where coalitions may break up. In Gomes and Jeheil, a coalition can move away from the efficient state to another state without the consent of some agents. They showed that if the non-consenting agent is not hurt by the transition from the efficient state to other states[1], the economy converges to the efficient state in a finite number of contracting rounds if players are sufficiently patient. Hyndman and Ray (2007) examined the effects of up-front transfers among the members of coalitions on convergence to the efficient state. They provide an example of all equilibrium paths remaining in the inefficient state. Bloch and Gomes (2007) considered the problem of commitment in players' actions, with players endogenously choosing whether to exit a bargaining game by committing to their actions. They showed that the externalities on the payoffs for exiting players become a source of inefficiency in equilibrium. In economies without externalities among coalitions (that is, in characteristic function form games), efficiency is achieved through voluntary bargaining with renegotiations, as shown by Siedmann and Winter (1998), Okada (2000), and Hyndman and Ray (2007). Maskin (2003) and Hafalir (2007) define the Shapley value and core in the partition function form game. They also provide the mechanism that implements the value, or core, and examine the inefficiency properties of coalition formation.

The main purpose of this paper is to investigate whether a slight restriction of the contracting process affects the result that the equilibrium path converges to the efficient state

[1] they say that "the efficient state is negative externalities-free."

in the framework of Gomes (2005). Instead of using the multilateral contracting process of Gomes, we consider the *bilateral contracting process*. Under bilateral contracting, only one integration between two coalitions is allowed through negotiations in each bargaining round. Note that our definition of bilateral contracting implies that there is contracting or bargaining between more than three players if either of the coalitions contains multiple players. In this sense, our restriction is very weak. Our attempt in this paper is the first step to examine the robustness of Gomes's result against the restriction of negotiation processes. There are some reasons for considering bilateral contracting. First, it is much easier to coordinate their actions between bilateral agents than between multilateral agents. The coordination problem causes bilateral contracting in the real world. Mergers of firms are often conducted bilaterally. There are cases in which multilateral negotiations are impossible. For example, if customers are located far apart, a salesperson should negotiate with each customer sequentially. Second, there are many economic applications in which the bargaining process is restricted bilaterally. Macho-Stadler, Perez-Castrillo and Porteiro (2006) considered coalition formation (merger) in a Cournot oligopolistic market with bilateral agreements in which only two of the existing coalitions can merge in any period. The key difference between free trade agreements (FTAs) and customs unions relates to whether negotiations about external and internal tariffs on each country's good are undertaken bilaterally or multilaterally. Yi (1996, 1997), Seidmann (2005), Goyal and Joshi (2005), Furusawa and Konishi (2007), and Agion, Antras and Helpman (2007) studied the formation of FTAs and customs unions in a model of international trade. In models of network formation, such as that of Jackson and Wolinsky (1996), in which the network comprises links between the players, the link between two players is an important factor. The process of forming links may be considered a bilateral contracting process.

We consider a bargaining situation described by a three-person strategic form game. We do not adopt the partition function form game in many previous studies (see, Bloch, 1996, Yi, 1996, Ray and Vohra, 1999, Maskin, 2003, Gomes, 2005, Diamantoudi and Xue, 2007, Hafalir, 2007, and Hyndman and Ray, 2007). The situation by the partition function form game can be considered as a special case of the strategic form game situation. Okada (2005) and Bloch and Gomes (2006) also consider a strategic form game as a primitive. The bargaining rule is simple. In every period, one player is randomly selected as a proposer with equal probability among all players. The player proposes a contract that specifies their actions and transfers for the members of the endogenously determined coalition. A coalition arises from the integration of two existing coalitions at the period. If all other players in the coalition accept the proposal, they can reach agreement on the contract. If some player in the coalition rejects the proposal, the contract is not renewed and the pre-existing contract remains valid. Then, under the agreed contracts, all players who have contracted follow their agreed-upon actions. Players without contracts select their actions independently. The per-period payoff is received by each player and the game goes to the next period and the same process is repeated with new proposers being randomly selected. In our bargaining model, a coalition represents a set of players who have made binding agreements about their actions and transfers. This notion is consistent with that in cooperative games in strategic form (see, for example, Aumann, 1961, 1967, and Shapley and Shubik, 1969). In this case, the state of the economy in each period is represented by the existing contracts about all players' actions and transfers. By contrast, in the partition function form game, a coalition is treated like a player who maximize a payoff. Members of the coalition are assumed to choose their actions to maximize their aggregate payoff. Moreover, it often be assumed that all coalitions choose the actions of their members simultaneously. The difference between a coalition in our setting and one in the partition function form game is derived from what the members of coalitions can agree on their actions and transfers hen forming a coalition. By adopting the strategic form game, we clarify the economic situations considered in the partition function form game.

This paper obtains the following results. When the members of coalitions make binding agreements about their actions and transfers in the round of coalition formation, almost all equilibrium paths converge to a Pareto efficient allocation and the grand coalition is formed. Bilateral contracting does not affect the convergence of the economy to the efficient state. In the noncooperative bargaining game model based on the partition function form game, which is corresponding to the model of Gomes (2005), the economy can remains in an inefficient state forever. The Coase theorem does not necessarily apply when the contracting processes are restricted to the bilateral contracting.

The rest of this paper is organized as follows. Section 2 provides the basic model and the solution concept. Section 3 characterizes equilibria in our noncooperative bargaining game. Section 4 gives an interpretation of the partition function form game in the bargaining game model based on the strategic form game and show the possibility of inefficient equilibrium in the bargaining game model with the partition function form game. Section 5 presents concluding remarks. All proofs are gathered in the Appendix.

2 The Model

We consider a bargaining game with three players. The set of players is given by $N = \{1, 2, 3\}$. We begin with the strategic form game $G = (N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N})$ as a primitive, where A_i is the (finite or compact) set of player i's actions and $u_i : \prod_{i\in N} A_i \to R$ is the payoff function for player i. Let us define the set of all action profile by $A = \prod_{i\in N} A_i$ and denote a generic element by $a \in A$. Then, $u_i(a)$ represents the flow of payoff per period for player i in an action profile $a \in A$. In the strategic form game G, the payoff of each player depends on not only his or her own action but also other player's actions. The interdependences express externalities among players. Segal (1999) defined externalities in the same way. The bargaining game based on a strategic form game contains the bargaining game model with externalities, such that Ray and Vohra (1999), Maskin (2003), Gomes (2005), Hyndman and Ray (2007), and Hafalir (2007). The discount factor per period is denoted by δ. We assume that payoffs are transferable among players. A subset S of N is called a *coalition* of players. In this paper, except in Section 4, a coalition is regard as the set of players who have made binding agreements about their actions and transfers between them. In other words, the set of players who have written contracts that specify their actions and monetary transfers among members conditional on the contracts of outside players means a coalition. The definition of coalition follows from that in Konishi and Ray (2003), Gomes and Jehiel (2005) and Okada (2005). It is natural in the cooperative game in strategic form game by Aumann (1961, 1967) and Shapley and Shubik (1969). Our definition is different from that in a partition function form game; Ray and Vohra (1999), Maskin (2003), Gomes (2005), Hafalir (2007), Hyndman and Ray (2007). In the partition function form game, agreements between the members of coalitions about their actions and transfers are not indicated explicitly. We will consider the partition function form game in Section 4. Transfers $t_S = (t_i)_{i\in S}$ among members of S satisfies $\sum_{i\in S} t_i = 0$[2]. A partition of N is referred as a coalition structure π. The set of all coalition structures is denoted by Π. When $N = \{1,2,3\}$, all coalition structures are as follows:

$$\pi_0 = \{\{1\},\{2\},\{3\}\},\ \pi_1 = \{\{1\},\{2,3\}\},\ \pi_2 = \{\{2\},\{1,3\}\},$$
$$\pi_3 = \{\{3\},\{1,2\}\},\ \pi_N = \{\{1,2,3\}\}.$$

The subscript of π_i, $i = 1,2,3$, indicates the player index of the singleton coalition in the coalition structure.

Our bargaining game is an infinite period game. Every period is divided to two different phases. Any period starts with a *contracting phase*. After the contracting phase, each player chooses an action $a_i \in A_i$ simultaneously. This phase is called an *action phase*. A coalition structure at period k is denoted by $\pi^k \in \Pi$. At the initial period (period 0), no players have been contracted. Thus, the game starts with the no-contracting state, i.e., $\pi^0 = \pi_0$. The precise rule of the bargaining game is given as follows.

(i) **Contracting phase**: At every period, a player is chosen at random (with equal probability). The selected player i proposes a contract to the members of a coalition including player i and one outside-coalition. The contract specifies a binding agreement about actions and monetary transfers. Thus, player i can propose to form a coalition $S \cup \hat{S}$ such that $S \in \pi^k$, where $i \in S$, and $\hat{S} \in \pi^k$. For all members of $S \cup \hat{S}\backslash\{i\}$, player i makes a proposal of actions $a_{S\cup\hat{S}} \in A_{S\cup\hat{S}}$ and transfers $t_{S\cup\hat{S}}$ satisfying $\sum_{j\in S\cup\hat{S}} t_j = 0$. We call this process a *bilateral contracting process*.

All other members in $S\cup\hat{S}$ either accept or reject the proposal sequentially according to predetermined order. If all players in $S\cup\hat{S}$ accept the proposal, then it is agreed upon and becomes a binding agreement. This means that a contract can be rewritten by unanimous consent. If some player rejects the proposal, no contract is renewed.

(ii) **Action phase**: In this phase, all players who have written contracts are bound to

[2]Alternatively, we can assume that $\sum_{i\in S} t_i \leq 0$ without affecting the results. The notations and proofs, however, are complicated.

follow their agreed actions. On the other hand, players without contract choose their individual actions independently. Then, each player recieves the per-period payoff and the game goes to the next period.

The feature of this paper is a restriction on forming coalitions. In every coalition structure, only two coalitions out of all existing coalitions can be integrated. In other words, bilateral contracting between two coalitions is allowed in every period. Nevertheless to say, if coalitions consist of multiple players, bilateral contracting implies contracting between multiple (more than three) players. By contrast, Gomes (2005) and Hyndman and Ray (2007) impose no restriction on the contracting process. In a game with three players, the direct transition from the coalition structure with singleton players to that with the grand coalition is prohibited. The second feature of this paper is to consider that coalitions can only expand and coalition structures become coarser because ongoing agreements are binding, but can be renegotiated with the unanimous consent. In Hyndman and Ray (2007), this coalition formation process is called an environment of *permanent agreements*. Gomes (2005) also considers the expansions of coalitions, whereas Gomes and Jehiel (2005) include the possibility that coalitions may break up.

The above bargaining model is represented by an infinite-length extensive form game with perfect information. We study the *Markov perfect equilibrium* (MPE). In a MPE, the strategies used by the players may depend only on the state of the game at the end of previous period or phase. Such strategies are called *Markov strategies*. We consider that the state in the beginning of the contracting phase for each period consists of the existing contracts. The proposals by proposers in a Markov strategy depend on the existing contracts at the beginning of the period. Of course, the response by each player depends on the proposals by proposers in a Markov strategy. Moreover, Markov strategies in the action phase depend only on existing contracts at the end of the contracting phase in the same period. Let $\sigma_i = \{\sigma_i^t\}_{t=0}^{\infty}$ be a strategy for player i and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ be a strategy combination. The t-th period strategy σ_i^t for player i specifies player i's action as follows: (i) when player i is a proposer in the contracting phase at period t, his or her action is a proposal about contracts, (ii) when i is a responder in the contracting phase at period t, his or her action is an element of {accept, reject}, and (iii) in the action phase, his or her action is $a_i \in A_i$.

Definition 1. A strategy combination $\sigma^* = (\sigma_i^*)_{i \in N}$ is called a *Markov perfect equilibrium* (MPE) if σ^* is a subgame perfect equilibrium where σ_i^* is a Markov strategy for each player i.

We focus on pure strategies. In many applications such as merger in the oligopoly market, research coalitions, public goods provision, customs unions and FTAs among others, a pure strategy equilibrium is focused on. By restricting pure strategy MPEs, we can maintain the clear correspondence between economic applications and the results in this paper. Because players' strategies are restricted, it becomes difficult to achieve efficient allocations. Thus, the Coase theorem is at a disadvantage in our setting.

Static efficiency in our model reduces to aggregate welfare efficiency since payoffs are assumed to be transferable. Then, we can define the following concept of efficiency.

Definition 2. An action profile $a^* \in A$ is called *(static) Pareto efficient* if and only if $a^* \in \arg\max_{a \in A} \sum_{i \in N} u_i(a_i)$.

In the grand coalition, players can choose the static Pareto efficient action profile a^*. Therefore, the coalition structure π_N containing the grand coalition under a^* is called *efficient state*. Our framework satisfies the *grand coalition efficiency* in Gomes (2005) and the *grand coalition superadditivity* (GCS) in Hyndman and Ray (2007) since $\sum_{i \in N} u_i(a^*) \geq \sum_{i \in N} u_i(a)$ for all $a \in A$. We should note that there is another kind of inefficiency in a dynamic game model. That is a delay of agreements.

The main concern of this paper is to examine whether the Pareto efficient action profile is eventually realized and whether a delay of agreements happens in a MPE. Gomes (2005) showed that in any multilateral contracting game where the grand coalition is efficient, a delay of agreements never happen and the equilibrium path converges to the efficient grand coalition after a finite number of period. We will check the robustness of Gomes's result against restricting the process of coalition formation to the bilateral contracting process.

3 Characterization of Equilibria

In this section, we characterize the Markov perfect equilibria under some additional assumptions. Let us firstly consider behaviors of each player in the action phase.

3.1 Behaviors in the action phase

If players have reached agreements about their actions by the preceding contracting phase, then they select their agreed-upon actions at the action phase. On the other hand, players who have not been made agreements in the contracting phase choose their actions independently. We can classify the equilibrium behaviors of players at the action phase into three categories.

No contract: Let us consider the case in which there is no binding agreements. The corresponding coalition structure is $\pi_0 = \{\{1\}, \{2\}, \{3\}\}$. In this case, all players select their actions independently. Each player is faced with a strategic form game $G = (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$ and plays a Nash equilibrium strategy of the strategic form game. We denote a Nash equilibrium of strategic form game G by $a^0 = (a_i^0)_{i \in N}$. Thus, every player i chooses action a_i^0 such that

$$u_i(a_i^0, a_{-i}^0) \geq u_i(a_i, a_{-i}^0) \quad \text{for all} \quad a_i \in A_i.$$

We assume the existence of a pure strategy Nash equilibrium of the strategic form game.

Assumption 1. There exists a pure strategy Nash equilibrium of the strategic form game $G = (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$.

Contracts between two players: Consider the case in which player j, k have reached agreements about actions, $\hat{a}_j$ and $\hat{a}_k$, in the previous contracting phase and player i has no contract. This implies that the coalition structure is π_i, $i = 1, 2, 3$. In this case, player j and k must choose action $\hat{a}_j$ and $\hat{a}_k$ in the action phase. Because player i know the contract between player j and k, player i select a solution of the maximization problem:

$$\max_{a_i \in A_i} u_i(a_i, \hat{a}_j, \hat{a}_k).$$

The best-response correspondence of player i is denoted by $r_i(\hat{a}_j, \hat{a}_k) = \text{argmax}_{a_i \in A_i} u_i(a_i, \hat{a}_j, \hat{a}_k)$. Then, player i's equilibrium action is $a_i \in r_i(\hat{a}_j, \hat{a}_k)$.

Contracts among all players: Binding agreements about actions for all players have been reached. The grand coalition is formed, i.e., the coalition structure is $\pi_N = \{\{1, 2, 3\}\}$. Each player i, $i = 1, 2, 3$, selects the agreed-upon action in the contracting phase.

In the contracting phase, each player negotiates the menu of contracts, taking into account of the above equilibrium behaviors at the action phase. Then, we analyze the reduced form in which the equilibrium behaviors in the action phase are embedded in the contracting phase. The behaviors in the action phase will not emerge explicitly from now on. We will focus on the contracting phase.

3.2 Behaviors in the contracting phase

In the contracting phase, each player negotiates contracts about actions and transfers for coalitions, given the equilibrium behaviors in the action phase. Then, we can reduce the per-period game with two phases to that only consisting of the contracting phase. In addition, because we focus on a Markov perfect equilibrium, then the Markov strategies in the contracting phase at period t depend only on the existing contracts which consists of the corresponding coalition structure $\pi \in \Pi$, the binding actions and transfers at period $t-1$. Because a coalition is the set of players who have written a contract among them, the coalition structure is derived from the existing contracts. The non-binding actions by singleton players in the action phase does not influence players' strategies in the contracting phase at the next period. We denote the *state* at period k by $(\pi^k; a^k, t^k)$. As a history of the game at period τ, a sequence of all past state until period τ is given by $(\pi^0; a^0, t^0), (\pi^1; a^1, t^1), \ldots, (\pi^{\tau-1}; a^{\tau-1}, t^{\tau-1})$. In a Markov perfect equilibrium, the strategies in the contracting phase at period τ depend only on the $\tau - 1$ period state $(\pi^{\tau-1}; a^{\tau-1}, t^{\tau-1})$.

We assume the strict order relation between the Pareto efficient action profile a^* and the Nash equilibrium action profile a^0 of G in the sense of aggregate efficiency.

Assumption 2. We assume that

$$\sum_{i \in N} u_i(a^*) > \sum_{i \in N} u_i(a^0).$$

Since we restrict the process of forming coalitions to bilateral contracting, the set of possible coalitions proposed by player i is also restricted. We denote by $\Psi_i(\pi_j)$ the set of possible coalitions proposed by player i in coalition structure π_j. Then, for $i, j, k = 1, 2, 3$, $i \neq j \neq k$,

$$\begin{aligned}
\Psi_i(\pi_0) &= \{\{i, j\}, \{i, k\}, \{i\}\}, \\
\Psi_i(\pi_i) &= \{\{i\}, \{i, j, k\}\}, \\
\Psi_i(\pi_j) &= \{\{i, k\}, \{i, j, k\}\}, \\
\Psi_i(\pi_N) &= \{\{i, j, k\}\}.
\end{aligned}$$

In addition, a proposer offers a contract which specifies the constant action a_S^τ and transfer t_S^τ in all subsequent periods $\tau = t, t+1, \ldots$ for the members of coalition S.

At $\tau = 0$ (the initial period), there is no contract. This means that the coalition structure is π^0. Moreover, each player plays a Nash equilibrium action a_i^0 in the action phase, and there is no transfer between players, i.e., $t^0 = (t_1^0, t_2^0, t_3^0) = (0, 0, 0)$. Thus, the game starts with state $(\pi^0; a^0, t^0)$.

Let σ be a MPE. For every state $(\pi; a, t)$ and player i, we denote by $\phi_i(\pi; a, t)$ the *expected equilibrium discounted payoff* of player i in σ when the game starts with state $(\pi; a, t)$. If the game is in state $(\pi; a, t)$ and if player i rejects the proposal, the continuation payoff of player i in σ is given by $x_i(\pi; a, t)$, where

$$x_i(\pi; a, t) = u_i(a) + t_i + \delta \phi_i(\pi; a, t).$$

Player i receives the flow of payoff of $u_i(a) + t_i$ for the current period, and since the game starts with state $(\pi; a, t)$ again in next period, he or she obtains the expected payoff of $\phi_i(\pi; a, t)$.

(i) coalition structure π_N: Let us start the characterization of MPE behaviors in the contracting phase at $(\pi_N; a^*, t)$, where a^* is a Pareto efficient action profile. The grand coalition N has already been formed. Therefore, unanimous agreements would be required to renew the contract.

Lemma 1. *In every pure strategy MPE at state $(\pi_N; a^*, t)$, every player proposes the status-quo; i.e., contract (a^*, t) is proposed for the grand coalition N.*

By Lemma 1, the expected equilibrium discounted payoff of player i at state $(\pi_N; a^*, t)$ is given by:

$$\phi_i(\pi_N; a^*, t) = \frac{1}{1-\delta}(u_i(a^*) + t_i), \ \ i = 1, 2, 3. \tag{1}$$

Next, we examine a MPE of the game starting from state $(\pi_N; a, t)$, where the action profile is not equal to the Pareto efficient action profile; $a \neq a^*$.

Lemma 2. *In every pure strategy MPE at state $(\pi_N; a, t)$ such that $a \neq a^*$, every player i proposes a solution of the following maximization problem in the first round:*

$$\begin{aligned} &\max_{\hat{a}, \hat{t}} u_i(\hat{a}) + \hat{t}_i + \delta\phi_i(\pi_N; \hat{a}, \hat{t}) \\ &\text{subject to } u_j(\hat{a}) + \hat{t}_j + \delta\phi_j(\pi_N; \hat{a}, \hat{t}) \geq x_j(\pi_N; a, t), \\ &\qquad\qquad u_k(\hat{a}) + \hat{t}_k + \delta\phi_k(\pi_N; \hat{a}, \hat{t}) \geq x_k(\pi_N; a, t). \end{aligned} \tag{2}$$

Moreover, the proposal is accepted. Player i's proposal in the MPE σ at $(\pi_N; a, t)$ is given by $(\hat{a}, \hat{t})$ such that $\hat{a} = a^*$ and $\hat{t}$ satisfying $\hat{t}_i = -\hat{t}_j - \hat{t}_k$,

$$\begin{aligned} &u_j(a^*) + \hat{t}_j + \frac{\delta}{1-\delta}(u_j(a^*) + \hat{t}_j) = x_j(\pi_N; a, t), \ \ \textit{and} \\ &u_k(a^*) + \hat{t}_k + \frac{\delta}{1-\delta}(u_k(a^*) + \hat{t}_k) = x_j(\pi_N; a, t). \end{aligned}$$

By definition of the game and Lemma 1, 2, the expected equilibrium discounted payoff of player i, $i = 1, 2, 3$, at (a, t, π_N) is given by

$$\phi_i(a, t, \pi_N) = \frac{1}{1-\delta}(u_i(a) + t_i) + \frac{1}{3}\frac{1}{1-\delta}\left[\sum_{j=1}^{3} u_j(a^*) - \sum_{j=1}^{3} u_j(a)\right].$$

It must be noted that the payoff allocation $(\phi_1(\pi_N; a, t), \phi_2(\pi_N; a, t), \phi_3(\pi_N; a, t))$ generated by a MPE σ is equal to the three-person Nash bargaining solution payoff allocation in the bargaining problem (U, d) where

$$\begin{aligned} U &= \left\{\frac{u_1(a)}{1-\delta}, \frac{u_2(a)}{1-\delta}, \frac{u_3(a)}{1-\delta} \;\middle|\; a \in A\right\}, \ \text{and} \\ d &= (d_1, d_2, d_3) = \left(\frac{u_1(a) + t_1}{1-\delta}, \frac{u_2(a) + t_2}{1-\delta}, \frac{u_3(a) + t_3}{1-\delta}\right). \end{aligned}$$

U is a set of feasible discounted payoff allocations and d is a disagreement point.

(ii) coalition structure π_i**:** Let us next go to coalition structure $\pi_i = \{\{i\}, \{j,k\}\}$, $i = 1, 2, 3$. Coalition $\{j,k\}$ has been formed in the previous period. Thus, a menu of actions a_j, a_k and transfers t_j, t_k has been agreed upon by player j and k. In these states, the transfers satisfy that $t_i = 0$ and $t_j + t_k = 0$. We assume the following tie-breaking condition:

Assumption 3. Each player proposes the larger coalition $\hat{S}$ if he or she obtains the same expected payoff in coalitions S and $\hat{S}$, $S \subsetneq \hat{S}$.

Under π_i, we can prove the following lemma.

Lemma 3. *In every pure strategy MPE at state* $(\pi_i; a, t)$*, every player* ℓ*,* $\ell = 1, 2, 3$*, proposes at the first round the solution of the maximization problem:*

$$\max_{(\hat{a},\hat{t},S)} u_\ell(\hat{a}) + \hat{t}_\ell + \delta\phi_\ell(\hat{\pi}; \hat{a}, \hat{t}) \tag{3}$$

$$\text{subject to } u_m(\hat{a}) + \hat{t}_m + \delta\phi_m(\hat{\pi}; \hat{a}, \hat{t}) \geq x_m(\pi_i; a, t), \text{for } m \in S, m \neq \ell,$$

$$\hat{a} = (\hat{a}_S, \hat{a}_{-S}), \text{where } \hat{a}_S \in A_S, \hat{a}_{-S} \in r_{-S}(\hat{a}_S), \text{ and } \hat{t} \in T_S,$$

$$S \in \Psi_\ell(\pi_i), \hat{\pi} \in \{\pi_i, \pi_N\}.$$

Moreover, the proposal is accepted. The equilibrium proposal of outside player i *is given as follows:* $\hat{a} = a^*$*,* $S = N$ *and* $\hat{t}$ *such that* $\hat{t}_i = -\hat{t}_j - \hat{t}_k$*,* $\hat{t}_j$ *and* $\hat{t}_k$ *satisfying*

$$u_j(a^*) + \hat{t}_j + \frac{\delta}{1-\delta}(u_j(a^*) + \hat{t}_j) = x_j(\pi_i; a, t), \text{and}$$
$$u_k(a^*) + \hat{t}_k + \frac{\delta}{1-\delta}(u_k(a^*) + \hat{t}_k) = x_k(\pi_i; a, t).$$

Note that every player i, j, k does not proposes the status quo or an unacceptable proposal in π_i. This means that no delay of agreements occurs when there is a coalition with multiple players. Lemma 3 says that if player i becomes a proposer, the economy arrives at the efficient state with a^* and the grand coalition. In addition, since A_j and A_k are finite (or compact), the economy could not stays at the inefficient state with coalition $\{j,k\}$ forever. Players j and k propose the grand coalition and the efficient action profile

a^* within finite periods[3]. Thus, the economy converges to the efficient state if it starts with π_i, $i = 1, 2, 3$.

(iii) coalition structure π_0: In coalition structure π_0, there is no contract and the game is in the initial state $(\pi_0; a^0, t^0)$.

Lemma 4. *In every pure strategy MPE at the initial state $(\pi_0; a^0, t^0)$, every player i proposes either the solution of the maximization problem:*

$$\max_{\hat{a}_S, \hat{t}_S, S} u_i(\hat{a}_S, \hat{a}_{-S}) + \hat{t}_i + \delta\phi_i(\pi_{N\setminus S}; (\hat{a}_S, \hat{a}_{-S}), (\hat{t}_S, 0)) \tag{4}$$

subject to

$$u_j(\hat{a}_S, \hat{a}_{-S}) + \hat{t}_j + \delta\phi_j(\pi_{N\setminus S}; (\hat{a}_S, \hat{a}_{-S}), (\hat{t}_S, 0)) \geq x_j(\pi_0; a^0, t^0),$$

$$j \in S, j \neq i, \hat{a}_S \in A_S, \hat{a}_{-S} \in r_{-S}(\hat{a}_S), \hat{t}_S \in T_S, S \in \Psi_i(\pi_0),$$

$$\text{where } \pi_{N\setminus S} = \pi_k \text{ if } S = \{i, j\}, \quad \pi_{N\setminus S} = \pi_j \text{ if } S = \{i, k\},$$

or the status quo (non-contracting state). If the solution of the maximization problem is chosen by player i, the proposal is accepted in σ, and if the status quo is chosen, the delay of agreements happens.

We should mention that the possibility of delayed agreements exists in the initial state $(\pi_0; a^0, t^0)$. If all players propose the non-contracting state in every period, the state $(\pi_0; a^0, t^0)$ continues forever. However, the following proposition shows that it does not happen in our bargaining game.

Proposition 1. *In the bargaining game, the equilibrium path converges to the efficient state $(\pi_N; a^*, t)$ for any δ with probability one under Assumption 1, 2 and 3.*

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[3]Formally, the expected payoff $\phi_j(\pi; a, t)$ and $\phi_k(\pi; a, t)$ are monotone increasing with a transition of the states. Because A_j and A_k are finite (or compact), these expected payoffs have each maximum with respect to the pair of actions in $A_j \times A_k$. Because $\phi_j(\pi; a, t)$ and $\phi_k(\pi; a, t)$ converge to these maximum within finite period (we can choose a convergence sequence of the expected payoffs to the maximum value), player j or k would obtain the maximum value within finite periods ($\pi_j(\pi_i; a, t)$ or $\pi_k(\pi_i; a, t)$ converges to the maximum value when A_j and A_k is compact). After such a state is achieved, each player proposes the grand coalition. Moreover, $\hat{a} = a^*$ in the solution of (3) when $\hat{\pi} = \pi_N$.

Let us give a sketch of the proof. By Lemma 1, Lemma 2 and Lemma 3, if any two-person-coalition would be formed in π_0, the grand coalition will be formed and the Pareto efficient action profile will be realized. Therefore, the economy stays at inefficient state $(\pi_0; a^0, t^0)$ if all players propose the status quo in π_0. But, every player i is better off by offering coalition $\{i, j\}$ and contract $(a_i^0, a_j^0, t_i^0, t_j^0)$. Thus, the strategies in which all players propose the status quo in π_0 could not be equilibrium.

Proposition 1 says that the restriction of forming coalitions to bilateral contracting does not affect the convergence to the efficient state. However, Lemma 4 shows that the possibility of the delay in agreements remains, in contrasts to the results in Gomes (2005).

4 Inefficiency and Externalities

4.1 Interpretation of the partition function game

Although we started with a strategic form game, almost all previous studies have adopted *the partition function form game* as a primitive to consider the economic environments with externalities (see, Bloch, 1996, Yi, 1996, Ray and Vohra, 1999, Maskin, 2003, Gomes, 2005, Macho-Stadler et al., 2006, Currarini, 2007, Diamantoudi and Xue, 2007, Hafalir, 2007, Hyndman and Ray, 2007). In this section, we give an interpretation of the partition function form game approach from the viewpoint of a strategic form game. There are many applications of the partition function form game as follows.

(1) Merger in the Cournot oligopoly (Yi, 1996, Ray and Vohra, 1999, Macho-Stadler, et al., 2006)

Consider a Cournot oligopoly with a linear inverse demand $P(Q) = A - Q$, where Q is the industry output. Each firm i produces homogeneous output with a constant marginal cost c. Thus, firm i's cost function is given by $C(q_i) = cq_i$, where q_i is firm i's output. Under a cartel structure (i.e., a coalition structure) $\pi_i = \{S_1, \ldots, S_r\}$, each cartel S_a chooses their output to maximize their joint profit $\sum_{j \in S_a}[(A - Q)q_j - cq_j]$. Then, a partition function with coalition structure π is defined by

$$v(S_a, \pi) = \frac{(A - c)^2}{(r + 1)^2}.$$

(2) Merger in the Bertrand oligopoly (Deneckere and Davidson, 1985)

Consider a oligopoly with differential goods and identical constant (marginal and average) cost of c. A demand function is given by

$$q_i = B - p_i - \gamma(p_i - \frac{1}{n}\sum_{j=1}^{n} p_j), \quad i = 1, \ldots, n,$$

where p_i is the price charged and q_i is the quantity demanded of firm i's good, and $\gamma > 0$ is a substitutability parameter. Under a coalition structure $\pi = \{S_1, \ldots, S_r\}$, the members of coalition S_a choose their prices to maximize $\sum_{j \in S_a}(p_j - c)q_j$. Then, a partition function under π for S_a is defined by the aggregate profit of coalition S_a in a Nash equilibrium under π.

(3) Research coalitions in the Cournot oligopoly (Kamien, Muller and Zang, 1992, Yi, 1997 and 1998, Yi and Shin, 2000)

The market structure is same as in application (1). Each firm has one unit of research asset. For a member of coalition S_a, $s_a = |S_a|$, the cost function under a new technology developed with s_a units of research assets is given by $c(q_i, S_a) = \mu(s_a)q_i$, where q_i is firm i's output and $\mu(\cdot)$ is decreasing function. After forming coalition structure $\pi = \{S_1, \ldots, S_r\}$, each firm faces with the Courot competition in the product market. The aggregate profit of the members of coalition S_a is, for $a = 1, \ldots, r$,

$$v(S_a, \pi) = \frac{(A - (n+1)\mu(s_a) + \sum_{j=1}^{r} s_j \mu(s_j))^2}{(n+1)^2} s_a.$$

These represent the corresponding partition function.

(4) Customs unions and FTAs (Yi, 1996 and 1997, Seidmann, 2005, Goyal and Joshi, 2006, Furusawa and Konishi, 2007, Agion, Antras and Helpman, 2007)

The economy consists of n countries. Each country produces one good at a constant marginal cost c in terms of the numeraire good. These goods are assumed to be differentiated. Country i's inverse demand function for country j's good is given by $p_{ij} = A - (1-\gamma)q_{ij} - \gamma Q_i$, where p_{ij} is a price of country j's product in country i's market, q_{ij} is country i's consumption of country j's product, $Q_i = \sum_{j=1}^{n} q_{ij}$, and γ is a substitution index between goods. Each country i's firm chooses the quantities in each country to maximize the profit. Country i selects a specific tariff τ_{ij} on imports from country j. Then, country j's effective marginal cost of exporting to country i is $c_{ij} = c + \tau_{ij}$. A customs union (a coalition) is defined as a group of countries with free trade among union members and a common external tariff to maximize the joint social welfare of members. FTA allows differential tariffs on external members. Under coalition structure $\pi = \{S_1, \dots, S_r\}$, $v(S_a, \pi)$ is given by the aggregate social welfare of members belonging to a customs union S_a.

(5) Public goods economy (Yi, 1997, Ray and Vohra, 1999 and 2001, Maskin, 2003, Gomes, 2007)

Each player i produces a pure public good z_i and has a utility given by $Z - c(z_i)$, where Z is the total amount of public goods, z_i is the quantity produced by the player, $c(z_i)$ is a cost to produce z_i. Coalition S_a, $s_a = |S_a|$, chooses its level of public good $Z_{S_a} = \sum_{j \in S_a} z_j$ to maximize $Z_{S_a} - c(Z_{S_a}/s_a)$. Under $\pi = \{S_1, \dots, S_r\}$, each coalition chooses Z_S simultaneously. Then, the aggregate payoff of the members of coalition S_a represents $v(S_a, \pi)$

(6) Common resource problem (Funaki and Yamato, 1999)

The aggregate payoff of the members of coalition S is given by

$$\frac{Z_S}{Z_N} f(Z_N) - qZ_S,$$

where $Z_S = \sum_{j \in S} z_j$, $S \subseteq N$, z_j is the amount of labor of fisherman j, and $f(Z_N)$ is the amount of fish caught by the total amount of labor Z_N. Each coalition chooses Z_S to maximize its aggregate payoff simultaneously.

Let us introduce the concept of Nash equilibrium under coalition structure[4].

Definition 3. An action profile $a^{**}(\pi_i)$ is called *a Nash equilibrium under coalition structure* π_i if

$$\begin{aligned} &u_i(a^{**}(\pi_i)) \geq u_i(a_i, a_j^{**}(\pi_i), a_k^{**}(\pi_i)), \text{ for all } a_i \in A_i, \\ &u_j(a^{**}(\pi_i)) + u_k(a^{**}(\pi_i)) \geq u_j(a_j, a_k, a_k^{**}(\pi_i)) + u_k(a_j, a_k, a_k^{**}(\pi_i)), \\ &\qquad\qquad\qquad\qquad \text{for all } a_j \in A_j, a_k \in A_k. \end{aligned}$$

[4]This concept is also called *coalitional equilibrium* in Ichiishi (1981) and Ray (2008)

If $\pi_i = \pi_0$, the definition consists with a Nash equilibrium a^0 in a strategic form game G.

Consider a three-player game $N = \{1, 2, 3\}$. By the above applications, we should interpret the partition function to be defined as follows:

$$\begin{aligned}
&v(\{i,j,k\};\pi_N) = u_i(a^*) + u_j(a^*) + u_k(a^*),\\
&v(\{j,k\};\pi_i) = u_j(a^{**}(\pi_i)) + u_k(a^{**}(\pi_i)),\\
&v(\{i\};\pi_i) = u_i(a^{**}(\pi_i)),\\
&v(\{i\};\pi_0) = u_i(a^0),\ \text{for } i = 1,2,3, i \neq j \neq k,
\end{aligned} \tag{5}$$

where a^* is a Pareto efficient action profile, $a^{**}(\pi_i)$ is a Nash equilibrium under π_i, and a^0 is a Nash equilibrium of the game G.

We add the following assumption.

Assumption 4. For $i, j, k = 1, 2, 3$, $i \neq j \neq k$,

$$u_i(a^{**}(\pi_i)) + u_j(a^{**}(\pi_i)) + u_k(a^{**}(\pi_i)) \geq u_i(a^0) + u_j(a^0) + u_k(a^0).$$

All above examples satisfy Assumption 4.

In the framework of our noncooperative bargaining game, the partition function form game is corresponding to the following game in which every period is divided to two phases:

(i') **Coalition formation phase:** In every period, a player is chosen at random. The selected player proposes a coalition. The members of the coalition sequentially accept or reject the proposal. If all members accept, the coalition is formed. The members of coalitions agree to behave as if they maximize their aggregate payoff in the action phase. However, they can not commit their actions in advance in the coalition formation phase. Transfers between the members of coalition are allowed in the action stage. Of course, only a merger between two coalitions is considered here.

(ii') **Action phase:** All coalitions simultaneously choose their actions to maximize their aggregate payoff under a given coalition structure.

In the bargaining game, the action profile is corresponding to each coalition structure as in (5).

Definition 4. We say that the noncooperative bargaining game is in *the partition function form game situation* if the per-period game consists of coalition formation phase (i') and action phase (ii') mentioned above.

Following the definition in Yi (1996), Maskin (2003), Currarini (2007) and Hafalir (2008), the notions of externalities in our model are defined as follows.

Definition 5. We say that the game has *negative externalities* if it holds that $\max_{a_k \in A_k} u_k(a_i^0, a_j^0, a_k) > \max_{a_k \in A_k} u_k(a_i^{**}(\pi_k), a_j^{**}(\pi_k), a_k)$ for all $\{i, j\}$, $i, j, k = 1, 2, 3$, $i \neq j \neq k$.

In the game with negative externalities, when coalitions merge to form a larger coalition, outside coalitions not involved in the merger are worse off. Among above examples, merger in the Bertrand competition, research coalitions in the Cournot oligopoly and customs unions and FTAs belong to the games with negative externalities.

Definition 6. We say that the game has *positive externalities* if it holds that $\max_{a_k \in A_k} u_k(a_i^0, a_j^0, a_k) < \max_{a_k \in A_k} u_k(a_i^{**}(\pi_k), a_j^{**}(\pi_k), a_k)$ for all $\{i, j\}$, $i, j, k = 1, 2, 3$, $i \neq j \neq k$.

In the game with positive externalities, if coalitions merge to form a larger coalition, outside coalitions are better off. The games with positive externalities contains merger in the Cournot competition, public goods economy, common resource problem, and a prisoners' dilemma game.

Definition 7. We say that the game has *no externalities* if it holds that $\max_{a_k \in A_k} u_k(a_i^0, a_j^0, a_k) = \max_{a_k \in A_k} u_k(a_i^{**}(\pi_k), a_j^{**}(\pi_k), a_k)$ for all $\{i, j\}$, $i, j, k = 1, 2, 3$, $i \neq j \neq k$

In the partition function form game situation, we can obtain the following proposition, which is corresponding to Proposition 1 in the bargaining game based on a strategic form game.

Proposition 2. *In the partition function form game situation, the equilibrium path converges to the efficient state for any δ with probability one if there is at least one (two-player) coalition $\{i, j\}$ satisfying condition:*

$$\begin{aligned} &[u_i(a^{**}(\pi_k)) - u_i(a^0)] + \frac{1}{3}\delta \left[\sum_{\ell=1}^{3} u_\ell(a^*) - \sum_{\ell=1}^{3} u_\ell(a^{**}(\pi_k)) \right] \\ &\quad + [u_j(a^{**}(\pi_k)) - u_j(a^0)] + \frac{1}{3}\delta \left[\sum_{\ell=1}^{3} u_\ell(a^*) - \sum_{\ell=1}^{3} u_\ell(a^{**}(\pi_k)) \right] > 0. \end{aligned} \tag{6}$$

Proposition 2 shows the possibility that the economy remains in the inefficiency state forever (with probability one) in the partition function form game situation. The game with negative externalities and with no externalities satisfy condition (6) under Assumption 4 because terms of $[u_i(a^{**}(\pi_k)) - u_i(a^0)]$ and $[u_j(a^{**}(\pi_k)) - u_j(a^0)]$ are always positive or equal to zero. Therefore, we have the following corollary of Proposition 2.

Corollary 1. *In the partition function form game situation with negative externalities and the game with no externalities, the equilibrium path converges to the efficient state for any δ with probability one.*

The restriction of forming coalitions to a bilateral merger does not affect the convergence to the efficient state in economies with negative externalities. On the other hand, the game with positive externalities does not necessarily satisfy condition (6) because the value of $([u_i(a^{**}(\pi_k)) - u_i(a^0)] + [u_j(a^{**}(\pi_k)) - u_j(a^0)])$ is negative.

Corollary 2. *The economy remains in the inefficient state $(\pi_0; a^0, t^0)$ forever only if the game is in the partition function form game situation with positive externalities.*

Corollary 2 says that the restriction of forming coalitions to the bilateral merger affects the convergence to the efficient state, in contrast to the result in Gomes (2007). Under the superadditivity of the partition function as in Maskin (2003), the value of $([u_i(a^{**}(\pi_k)) - u_i(a^0)] + [u_j(a^{**}(\pi_k)) - u_j(a^0)])$ is positive. Then, this kind of inefficiency could not happen in Maskin's paper. Ray and Vohra (1997), Yi (1997), Maskin (2003), Diamantoudi and Xue (2007) have already shown that the grand coalition does not necessarily formed under positive externalities from different viewpoints. Corollary 2 indicates the similar tendency as the result in previous studies.

4.2 Examples

4.2.1 Inefficient equilibrium

We present an example in which the economy stays with the inefficient state $(\pi_0; a^0, t^0)$ forever in equilibrium. By Corollary 2, the game must be in the partition function form game situation with positive externalities. Let us consider mergers in the Cournot competition. The inverse demand function is given by $P(Q) = \alpha - \beta Q$, where $Q = \sum_{i=1}^{3} q_i$, and firm i's cost function is $c(q_i) = cq_i$. We normalize $(\alpha - c)^2/\beta = 1$. A per-period profit of firm i at action profile a is denoted by $u_i(a)$. Under cartel structure π_0, it holds that $v(\{i\};\pi_0) = u_1(a^0) = u_2(a^0) = u_3(a^0) = 1/16$. Under π_k, $k = 1,2,3$, $i \neq j \neq k$, we have that $v(\{i,j\};\pi_k) = u_i(a^{**}(\pi_k)) + u_j(a^{**}(\pi_k)) = 1/9$ and $v(\{k\};\pi_k) = u_k(a^{**}(\pi_k)) = 1/9$. Moreover, $v(\{1,2,3\};\pi_N) = u_1(a^*) + u_2(a^*) + u_3(a^*) = 1/4$. Figure 1 summarizes the values of the corresponding partition function for π_0, π_3, π_N.

Figure 1.

	1	2	3	total
π_0	1/16	1/16	1/16	3/16
π_3	1/9		1/9	2/9
π_N	1/4			1/4

It is easy to see that condition (6) is not satisfied at $\delta < 3/4$ because

$$[u_1(a^{**}(\pi_3)) - u_2(a^0)] + [u_2(a^{**}(\pi_3)) - u_2(a^0)] = (1/9 - 1/8) = -1/72,$$
$$\text{and} \quad (2/3)\delta[\sum_{\ell=1}^{3} u_\ell(a^*) - \sum_{\ell=1}^{3} u_\ell(a^{**}(\pi_3))] = \delta(1/54).$$

We have the MPE strategies of the subgame starting from state $(\pi_N; a^*, t)$ and the subgame from state $(\pi_i; a^{**}(\pi_i), t)$ by Lemma 1 and Lemma 3' in the Appendix. Therefore, it is sufficient to construct a MPE strategy for each player at the initial state $(\pi_0; a^0, t^0)$. We show that the following strategies at the initial state are a part of a MPE of the bargaining game: All player, player 1, 2 and 3, make an unacceptable proposal, and they accept any proposal such that its discounted payoff is greater than $(1/(1-\delta))(1/16)$. By this strategy combination, each player obtains the expected payoff v_i of $(1/(1-\delta))(1/16)$.

It is enough to examine player 1's strategy because all players are symmetric. If player 1 proposes an acceptable proposal for coalition $\{1,2\}$, he or she obtains the discounted payoff:

$$\frac{1}{1-\delta}\frac{1}{9} + \frac{\delta}{1-\delta}\frac{2}{3}\left(\frac{1}{4} - \frac{2}{9}\right) - \left(\frac{1}{16} + \delta v_2\right). \tag{7}$$

Since $v_2 = (1/(1-\delta))(1/16)$, (7) is strictly less than $(1/(1-\delta))(1/16)$ if $\delta < 3/4$. We can apply the same argument to coalition $\{1,3\}$. Therefore, making an acceptable proposal is not optimal for player 1 when $\delta < 3/4$. Because all players make an unacceptable proposal, the economy remains the inefficient state $(\pi_0; a^0, t^0)$ in equilibrium.

Let us show that no inefficient equilibrium exist in our bargaining game based on a strategic form game in the above example. Every selected player can propose all feasible actions for the members of two-player coalition and, if the proposal is agreed, the members commit their actions in the action phase when forming coalitions. A singleton player on the outside of coalition chooses his or her action after observing the pre-committed actions of the members of coalition. The members of coalition can behave like a Stackelberg leader and the outside player does like a Stackelberg follower. In this example, the members of coalition $\{i, j\}$ can commit a pair of the Stackelberg leader's action, (a_i^0, a_j^0), at the contracting phase when they form the coalition. Moreover, player k responds by choosing the Stackelberg follower's action a_k^0 in the action phase. Then, under these actions, the members of coalition $\{i, j\}$ obtain their aggregate payoff of 1/8 and player k does the payoff of 1/16 in the period. When the grand coalition is formed, every proposer chooses a^*. The case of $k = 3$ is summarized in Figure 2.

Figure 2.

	1	2	3	total
π_0	1/16	1/16	1/16	3/16
π_3	1/8		1/16	3/16
π_N	1/4			1/4

Even if the members of coalition $\{i, j\}$ share the per-period payoff equally and propose the grand coalition under π_k, the proposer obtains the discounted payoff of

$$\frac{1}{1-\delta}\frac{1}{16} + \frac{1}{3}\frac{\delta}{1-\delta}\left(\frac{1}{4} - \frac{3}{16}\right).$$

Note that in the case that every player makes an unacceptable proposal, each player gets the discount payoff of $(1/(1-\delta))(1/16)$. The above acceptable proposal payoff-dominates making an unacceptable proposal. This means that some player makes some acceptable proposal in the initial state. Then, the economy converges to the efficient state in our bargaining game model based on a strategic form game.

4.2.2 Inefficient coalitional behaviors

Next, we give an example to see the difference between the bargaining game in strategic form games and that in partition function form game. Let us consider a variation of the prisoners' dilemma game. Player 1 and 2 have two actions, C (cooperate) or D (defect). Player 3 has only a single action a_3. The strategic form game is given as follows:

Figure 3.

	C	D
C	$2, 2, 0$	$-4, 3, -7$
D	$3, -4, -7$	$0, 0, 3$

Player 1, 2 has the dominant action D and the game has a unique Nash equilibrium (D, D, a_3). The negotiation starts with $a^0 = (D, D, a_3)$ and coalition structure π_0. We assume that coalition $\{i, 3\}$, $i = 1, 2$, is allowed to choose only a pair of actions (C, a_3). In addition, coalition $\{1, 2\}$ cannot change their Pareto efficient pair of actions (C, C) to others once they have chosen (C, C). The Pareto efficient action profile a^* is (C, C, a_3). Under these assumptions, the following strategy combination becomes a pure strategy MPE when $\delta > 5/8$. First, responders accept any proposals iff they can get the expected payoff greater than their continuation payoff. Moreover, every proposer extracts all surplus after giving the members of coalitions just their continuation payoffs. See Lemma 1, 2, 3 and 4 in detail. Therefore, we only mention the equilibrium proposals about coalitions and action profiles in each state. (a) In state $(\pi_N; a^*, t)$, all players propose the status quo. If $a \neq a^*$, every player proposes a Pareto efficient action profile a^* immediately. (b) In state $(\pi_1; (C, a), (t_2, t_3))$ or $(\pi_2; (C, a_3), (t_1, t_2))$, all players propose the grand coalition and a^*. (c) In state $(\pi_3; (C, C), (t_1, t_2))$, $(\pi_3; (C, D), (t_1, t_2))$ or $(\pi_3; (D, C), (t_1, t_2))$, every proposer makes an offer of the grand coalition and a^*. In state $(\pi_3; (D, D), (t_1, t_2))$, player 1, 2 proposes the present coalition $\{1, 2\}$ and a pair of actions (C, D), but player 3 proposes the grand coalition and a^*. (d) In the initial state $(\pi_0; a^0, t^0)$, player 1, 2 as a proposer proposes coalition $\{1, 2\}$ and (C, D). Player 3 makes an unacceptable proposal.

Here, we only show that the above strategies (d) in the initial state are (local) optimal because we can check the optimality of the strategies (a), (b), (c) in the same way. Under the above strategy combination, the expected discounted payoff for player 1, player 2 and player 3 are given by

$$\phi_1(\pi_0; a^0, t^0) = \phi_2(\pi_0; a^0, t^0) = \frac{8\delta - 1}{(3-\delta)(1-\delta)}, \phi_3(\pi_0; a^0, t^0) = \frac{5\delta - 11}{(3-\delta)(1-\delta)}.$$

By proposing $\{1, 2\}$ and (C, D), player 1 can get the expected discounted payoff of

$$-\frac{1}{1-\delta} + \frac{\delta}{1-\delta}\frac{2}{3}12 - \delta\phi_2(\pi_0; a^0, t^0) \tag{8}$$

when he or she is a proposer. Player 1 has alternatives; coalition $\{1, 2\}$ and a pair of actions (C, C), $\{1, 2\}$ and (D, D), or $\{1, 3\}$ and (D, a_3). Among these alternatives, a pair of $\{1, 2\}$ and (C, C) gives the highest expected payoff to player 1. The expected payoff is

$$2 + \hat{t}_1 + \frac{\delta}{1-\delta}(2 + \hat{t}_1), \text{ where } \hat{t}_1 = \frac{8\delta^2 + \delta - 6}{3 - \delta}. \tag{9}$$

If $\delta > 5/8$, we have that (8) $>$ (9). Making an unacceptable offer, player 3 obtain the expected payoff as

$$3 + \delta\phi_3(\pi_0; a^0, t^0). \tag{10}$$

If he or she makes an acceptable proposal of coalition $\{2, 3\}$ (or $\{1, 3\}$) and (C, a), player 3 can get the expected payoff as follows:

$$-7 + \hat{t}_3 + \delta\phi_3(\pi_1; (C, a), (-\hat{t}_3, \hat{t}_3)), \text{ where } \hat{t}_3 = \frac{-12\delta^2 + 17\delta - 12}{3 - \delta}. \tag{11}$$

It is satisfied that (10) $>$ (11) for all $0 \leq \delta < 1$.

Note that in equilibrium, the delay of agreements occurs when player 3 is selected as a proposer. In addition, the members of coalition $\{1, 2\}$ choose inefficient action profile (C, D), which yields the aggregate payoff of -1, on the equilibrium path, although they could select action profile (C, C) which generates the aggregate payoff of 4. When an action profile (C, D) is chosen by $\{1, 2\}$, outside player (player 3) is worse off because he or she incurs losses of -7. The bargaining position of player 1 and 2 in the future negotiation is improved, compared to player 3. Thus, they lose the current payoff, but they can get the future payoff. The above behaviors of coalition $\{1, 2\}$ can be interpreted as natural behaviors to seek the future payoff. Then, if the discount factor is large enough ($\delta > 5/8$), these behaviors would be equilibrium. On the other side, coalition $\{1, 2\}$ is assumed to take action pair (C, C) in the partition function form game. Such above strategic behaviors by coalitions in the bargaining game would be excluded under the partition function form game situations.

5 Conclusion

This paper examines whether the restriction from the multilateral contracting processes to the bilateral contracting ones affects the convergence to the efficient state. We showed that if the members of coalitions make binding agreements among their actions and transfers, almost all economies converge to an efficient state and the grand coalition is formed. On the other hand, if the members of coalitions only agree to behave to maximize their aggregate payoff and can not commit their actions against the outsiders, there is the possibility that the economy may stay in an inefficient state. In particular, inefficiency happens only in the case of positive externalities. This paper suggests that the efficiency result depends on whether the members of coalitions make binding agreements about their actions or not. Starting from the strategic form game clarifies relationships between players' actions and agreements. We should consider what coalitions can do in each economic applications. The reconsiderations of applications, especially, the network formation in Jackson and Wolinsky (1996), from the viewpoint of bilateral contracting could be an interesting research task. Moreover, the extensions to n-person games and incomplete information games remain in future work.

Appendix

Proof of Lemma 1: In the state $(\pi_N; a^*, t)$, if player i makes unacceptable proposals when he or she is a proposer and rejects all proposals when he or she is a responder, player i obtains the discounted payoff of $(u_i(a^*) + t_i)/(1 - \delta)$. Because $\phi_i(\pi_N; a^*, t)$ is corresponding to the discounted payoff for player i by his or her equilibrium strategy, then it is satisfied that

$$\phi_i(\pi_N; a^*, t) \geq \frac{1}{1-\delta}(u_i(a^*) + t_i), \ i = 1, 2, 3. \tag{12}$$

This implies that, because $t_1 + t_2 + t_3 = 0$,

$$\sum_{\ell=1}^{3} \phi_\ell(\pi_N; a^*, t) \geq \frac{1}{1-\delta} \sum_{\ell=1}^{3} u_\ell(a^*). \tag{13}$$

By the feasibility constraints, the aggregate amounts of (per-period) payoffs for all players is less than or equal to $\sum_{\ell=1}^{3} u_i(a^*)$. Then, we have that

$$\sum_{\ell=1}^{3} \phi_\ell(\pi_N; a^*, t) \leq \frac{1}{1-\delta} \sum_{\ell=1}^{3} u_\ell(a^*). \tag{14}$$

It follows from (13) and (14) that

$$\sum_{\ell=1}^{3} \phi_\ell(\pi_N; a^*, t) = \frac{1}{1-\delta} \sum_{\ell=1}^{3} u_\ell(a^*). \tag{15}$$

Because the coalition structure is π_N, player i have to get unanimous agreements among all the players (player j and k) in order to renew the contract. The continuation payoff of player j (that of player k) are represented by $x_j(\pi_N; a^*, t)$ ($x_k(\pi_N; a^*, t)$), respectively. Then, player j (k) accepts a proposal about his or her discounted payoff y_j (y_k) if and only if $y_j \geq x_j(\pi_N; a^*, t)$ ($y_k \geq x_k(\pi_N; a^*, t)$). Therefore, if player i makes an acceptable proposal, his or her optimal proposal is given by a solution of the maximization problem:

$$\begin{aligned} &\max_{\hat{a}, \hat{t}} u_i(\hat{a}) + \hat{t}_i + \delta\phi_i(\pi_N; \hat{a}, \hat{t}) \\ &\text{subject to } u_j(\hat{a}) + \hat{t}_j + \delta\phi_j(\pi_N; \hat{a}, \hat{t}), \\ &\qquad\qquad u_k(\hat{a}) + \hat{t}_k + \delta\phi_k(\pi_N; \hat{a}, \hat{t}), \quad \hat{a} \in A_N, \hat{t} \in T. \end{aligned} \tag{16}$$

Suppose that $\phi_i(\pi_N; a^*, t) > (u_i(a^*) + t_i)/(1-\delta)$ for some $i \in N$. By (12), we have

$$\sum_{\ell=1}^{3} \phi_\ell(\pi_N; a^*, t) > \frac{1}{1-\delta} \sum_{\ell=1}^{3} u_\ell(a^*).$$

This contradicts (15). Then, $\phi_i(\pi_N; a^*, t) = (u_i(a^*) + t_i)/(1-\delta)$ for all $i \in N$. By definition of the continuation discounted payoff, we also have $x_j(\pi_N; a^*, t) = (u_j(a^*) + t_j)/(1-\delta)$ and $x_k(\pi_N; a^*, t) = (u_k(a^*) + t_k)/(1-\delta)$. Then, it is easy to see that the solution of the problem (16) is $\hat{a} = a^*$ and $\hat{t} = t$. The corresponding discounted payoff of player i is given by $u_i(a^*) + t_i + \delta\phi_i(\pi_N; a^*, t)$.

On the other hand, even if player i makes an unacceptable proposal, he or she obtain the same discounted payoff $u_i(a^*) + t_i + \delta\phi_i(\pi_N; a^*, t)$. Therefore, whichever an acceptable or unacceptable proposal player i proposes, the status quo $(\pi_N; a^*, t)$ continues to next period.

Proof of Lemma 2: Let m_i be the maximum value of the problem (2). Because $t_i = -t_j - t_k$ and the objective function of the problem (2) is strictly increasing with t_i, it is satisfied at the solution $(\hat{a}^*, \hat{t}^*)$ of the problem that

$$u_j(\hat{a}^*) + \hat{t}_j^* + \delta\phi_j(\pi_N; \hat{a}^*, \hat{t}^*) = x_j(\pi_N; a, t),$$
$$u_k(\hat{a}^*) + \hat{t}_k^* + \delta\phi_k(\pi_N; \hat{a}^*, \hat{t}^*) = x_k(\pi_N; a, t).$$

If player i proposes the status quo (a,t) or an unacceptable proposal, then player i obtains the discounted payoff $x_i(\pi_N;a,t)$, giving the payoff of $x_j(\pi_N;a,t)$ to player k and the payoff of $x_k(\pi_N;a,t)$ to player k. Thus, $m_i \geq x_i(a,t,\pi_N)$.

By definition of $x_i(\pi_N;a,t)$ for all $i \in N$ and Assumption 2, we have that

$$\begin{aligned} x_i(\pi_N;a,t) &= \sum_{\ell=1}^{3} u_\ell(a) + \delta \sum_{\ell} \phi_\ell(\pi_N;a,t) - x_j(\pi_N;a,t) - x_k(\pi_N;a,t) \\ &< \frac{1}{1-\delta} \sum_{\ell=1}^{3} u_i(a^*) - x_j(\pi_N;a,t) - x_k(\pi_N;a,t). \end{aligned}$$

This implies that player i would not make an unacceptable proposal. In addition, the constraints in the maximization problem (2) are binding at the solution. Then, the objective function of the problem is given by

$$\sum_{\ell=1}^{3} u_\ell(\hat{a}) + \delta \sum_{\ell=1}^{3} \phi_\ell(\pi_N;\hat{a},\hat{t}) - x_j(\pi_N;a,t) - x_k(\pi_N;a,t).$$

Since $\sum_{\ell=1}^{3} u_{\ell=1}^{3}(\hat{a}) < \sum_{\ell=1}^{3} u_{\ell=1}^{3} u_\ell(a^*)$ (by Assumption 2) and $\delta \sum_{\ell=1}^{3} \phi_\ell(\pi_N;\hat{a},\hat{t}) \leq \delta \sum_{\ell=1}^{3} u_\ell(a^*)/(1-\delta)$ (by the feasibility contraint), it is easy to see that the solution of the problem is a pair of $(a^*,\hat{t}^*)$, where $\hat{t}_i^* = -\hat{t}_j^* - \hat{t}_k^*$ and $\hat{t}_j^*$, $\hat{t}_k^*$ satisfying

$$\begin{aligned} u_j(a^*) + \hat{t}_j + \delta\phi_j(\pi_N;a^*,\hat{t}^*) &= x_j(\pi_N;a,t), \\ u_k(a^*) + \hat{t}_k^* + \delta\phi_k(\pi_N;a^*,\hat{t}^*) &= x_k(\pi_N;a,t). \end{aligned}$$

Proof of Lemma 3: Firstly let us consider the case in which player i is selected as a proposer under the coalition structure $\pi_i = \{\{i\},\{j,k\}\}$.

If player i proposes coalition $\{i\}$ or an unacceptable proposal, the game continues to the next period in the same state $(\pi_i;a,t)$ and player i obtains the expected discounted payoff $u_i(a)+\delta\phi_i(\pi_i;a,t) = x_i(\pi_i;a,t)$. If player i makes an acceptable proposal for coalition $\{i,j,k\}$, he or she proposes the solution of the maximization problem:

$$\begin{aligned} &\max_{\hat{a},\hat{t}} u_i(\hat{a}) + \hat{t}_i + \delta\phi_i(\pi_N;\hat{a},t) \\ &\text{subject to } u_j(\hat{a}) + \hat{t}_j + \delta\phi_j(\pi_N;\hat{a},t) \geq x_j(\pi_i;a,t), \\ &\qquad u_k(\hat{a}) + \hat{t}_k + \delta\phi_k(\pi_N;\hat{a},t) \geq x_k(\pi_i;a,t). \end{aligned}$$

By the monotonicity of the objective function and Assumption 2, the solution of the above problem is given by $\hat{a} = a^*$ and $\hat{t}$ such that $\hat{t}_i = -\hat{t}_j - \hat{t}_k$ and $\hat{t}_j, \hat{t}_k$ satisfying

$$\begin{aligned} u_j(a^*) + \hat{t}_j + \delta\phi_j(\pi_N;a^*,t) &= x_j(\pi_i;a,t), \\ u_k(a^*) + \hat{t}_k + \delta\phi_k(\pi_N;a^*,t) &= x_k(\pi_i;a,t). \end{aligned}$$

Then, the maximum value of the problem is given by

$$\frac{1}{1-\delta}\sum_{\ell=1}^{3} u_\ell(a^*) - x_j(\pi_i;a,t) - x_k(\pi_i;a,t).$$

By Assumption 2, the maximum value is greater than or equal to $x_i(\pi_i;a,t)$. This implies that player i makes immediately an acceptable proposal in equilibrium and the proposal is accepted.

Next let us consider the case in which player j is selected as a proposer in the coalition structure π_i. By the rule of the game, three cases are possible: (i) player j proposes the status quo for coalition $\{j,k\}$, (ii) player j makes an unacceptable proposal for coalition $\{i,j,k\}$, and (iii) player j makes an acceptable proposal for coalition $\{j,k\}$ or $\{i,j,k\}$. In the case of (i) and (ii), player j obtains the expected payoff of $u_j(a) + t_j + \delta\phi_j(\pi_i;a,t)$, i.e., $x_j(\pi_i;a,t)$.

In the case of (iii), player j proposes the solution of the maximization problem (3) and the proposal is accepted(see, for example, the proofs of Theorem 1 in Okada, 1996, and Theorem 2.1 in Ray and Vohra, 1999). Let us show that player j (also k) does not propose the status quo or an unacceptable proposal. If player j proposes an unacceptable proposal or the status quo, he or she obtains the expected payoff $x_j(\pi_i;a,t)$. On the other hand, if player j offer the grand coalition N, the action profile a^* and transfers $\hat{t}^*$ such that $\hat{t}^*_j = -\hat{t}^*_k - \hat{t}^*_i$ and

$$u_k(a^*) + \hat{t}^*_k + \delta\phi_k(\pi_N;a^*,\hat{t}^*) = x_k(\pi_i;a,t),$$
$$u_i(a^*) + \hat{t}^*_i + \delta\phi_i(\pi_N;a^*,\hat{t}^*) = x_i(\pi_i;a,t),$$

player j get the expected payoff of $u_j(a^*) + \hat{t}^*_j + \delta\phi_j(\pi_N;a^*,\hat{t}^*)$ since the above proposal is accepted by player k and i. By Lemma 1 about $\phi_j(\pi_N;a^*,\hat{t}^*)$ and $\hat{t}^*_j = -\hat{t}^*_k - \hat{t}^*_i$, we can obtain

$$u_i(a^*) + \hat{t}^*_j + \delta\phi_j(\pi_N;a^*,\hat{t}^*) = \frac{1}{1-\delta}\sum_{\ell=1}^{3} u_\ell(a^*) - x_k(\pi_i;a,t) - x_i(\pi_i;a,t).$$

By definition of the continuation payoff and Assumption 2, we have

$$\begin{aligned}
&x_i(\pi_i;a,t) + x_j(\pi_i;a,t) + x_k(\pi_i;a,t)\\
&= (u_i(a) + u_j(a) + u_k(a)) + \delta(\phi_i(\pi_i;a,t) + \phi_j(\pi_i;a,t) + \phi_k(\pi_i;a,t)\\
&\leq \sum_{\ell}^{3} u_\ell(a^*) + \frac{\delta}{1-\delta}\sum_{\ell=1}^{3} u_\ell(a^*) = \frac{1}{1-\delta}\sum_{\ell=1}^{3} u_\ell(a^*).
\end{aligned}$$

Then, from the last equation and inequality,

$$u_j(a^*) + \hat{t}^*_j + \delta\phi_j(\pi_N;a^*,\hat{t}^*) \geq x_j(\pi_i;a,t).$$

By Assumption 3, player j proposes a^*, N and $\hat{t}^*$ rather than an unacceptable proposal. Moreover, let us denote the solution of problem (3) by $(\hat{a},\hat{t},\hat{S})$ and the induced coalition structure by $\hat{\pi}$. Then, the expected payoff for player j as a proposer is

$$u_j(\hat{a}) + \hat{t}_j + \delta\phi_j(\hat{\pi},\hat{a},\hat{t}) \geq u_j(a^*) + \hat{t}^*_j + \delta\phi_j(\pi_N;a^*,t) \geq x_j(\pi_i;a,t).$$

Because player j can get at least $x_j(\pi_i; a, t)$ as a responder, then $\phi_j(\hat{\pi}; \hat{a}, \hat{t}) \geq \phi_j(\pi_i; a, t)$. The above discussion shows that player j proposes the solution of the problem (3) at the first round (with no delay of agreements) and the proposal is accepted.

Proof of Lemma 4: From the solution of the problem (4), we can obtain the maximum discounted payoff for player i among acceptable proposals. At the solution, it is satisfied that in the case of $S = \{i, j\}$,

$$x_j(\pi_0; a^0, t^0) = u_j(\hat{a}_S, \hat{a}_{-S}) + \hat{t}_j + \delta\phi_j(\hat{\pi}; (\hat{a}_S, \hat{a}_{-S}), (\hat{t}_S, 0)).$$

On the other hand, if player i makes an unacceptable proposal, he or she obtains the discounted payoff $x_i(\pi_0; a^0, t^0)$. Because the grand coalition can not be proposed in π_0, the pair of payoffs $(x_i(\pi_0; a^0, t^0)$, $x_j(\pi_0; a^0, t^0))$ is not necessarily feasible under the constraints of the problem (4). Thus, we can not exclude the possibility of unacceptable proposals in π_0.

Proof of Proposition 1: From Lemma 1, 2 and 3, the economy converges to the efficient state (a^*) with probability one if some coalition is formed in the initial state since player i proposes the grand coalition N and action profile a^* and, then, the proposal is accepted. Therefore, only if all players propose an unacceptable proposal in the initial state, then the economy stays in the inefficient state ($a \neq a^*$). Let us give a proof by contradiction. Suppose that all players make an unacceptable proposal. Then, player i obtains the expected discounted payoff of $u_i(a^0)/(1-\delta)$ if he or she also proposes an unacceptable proposal. Under these strategies, player j's continuation payoff is $u_j(a^0)/(1-\delta)$. However, player i can offer coalition $\{i, j\}$ and (a^0, t^t). This proposal is accepted by player j because player j gets $u_j(a^0)/(1-\delta)$ as a expected payoff. Thus, the state $(\pi_k; a^0, t^0)$ would be realized. If player i (or j) proposes the grand coalition N and the efficient action profile a^* and $\hat{t}$ satisfying

$$\begin{aligned} u_j(a^*) + \hat{t}_j + \delta\phi_j(\pi_N; a^*, \hat{t}) &= x_j(\pi_k; a^0, t^0), \\ u_k(a^*) + \hat{t}_k + \delta\phi_k(\pi_N; a^*, \hat{t}) &= x_k(\pi_k; a^0, t^0), \end{aligned}$$

this proposal is accepted. Then, player i obtains the expected payoff:

$$u_i(a^*) + \hat{t}_i + \delta\phi_i(\pi_N; a^*, \hat{t}) = \frac{1}{1-\delta}\sum_{\ell=1}^{3} u_\ell(a^*) - x_j(\pi_k; a^0, t^0) - x_k(\pi_k; a^0, t^0).$$

By Assumption 2, we have

$$x_i(\pi_k; a^0, t^0) + x_j(\pi_k; a^0, t^t) + x_k(\pi_k; a^0, t^0) < \frac{1}{1-\delta}\sum_{\ell=1}^{3} .u_\ell(a^*).$$

Thus, the expected payoff for player i, $u_i(a^*) + \hat{t}_i + \delta\phi_i(\pi_N; a^*, \hat{t})$, is greater than the continuation payoff $x_i(\pi_k; a^0, t^0)$ in state $(\pi_k; a^0, t^0)$ when he or she is a proposer. Moreover, player i could make an unacceptable offer at all times and reject all proposals in $(\pi_k; a^0, t^0)$. Player i obtains $u_i(a^0)/(1-\delta)$ as a expected discounted payoff. Therefore, the continuation payoff $x_i(\pi_k; a^0, t^0)$ is greater than or equal to

$u_i(a^0)/(1-\delta)$. By the rule of game, it holds that

$$\phi_i(\pi_k; a^0, t^0) > x_i(\pi_k; a^0, t^0) \geq \frac{1}{1-\delta} u_i(a^0). \tag{17}$$

Applying the same discussion to player j, we can obtain

$$\phi_j(\pi_k; a^0, t^0) > \frac{1}{1-\delta} u_j(a^0). \tag{18}$$

Let us return to the initial state $(\pi_0; a^0, t^0)$. Suppose that player i proposes coalition $\{i, j\}$, action pair (a_i^0, a_j^0) and no transfer. It follows from (18) that the proposal is accepted by player j. As a result, player i obtains his or her payoff of $u_i(a^0) + \delta\phi_i(\pi_k; a^0, t^0)$. By (17), the payoff is greater than $u_i(a^0)/(1-\delta)$. Thus, the strategy combination could not be optimal such that all players make an unacceptable proposal at the initial state. This is a contradiction.

Proof of Proposition 2: In the partition function form game situation, the action profile is fixed by a^* under coalition structure π_N. Since the bargaining procedure is same, we can repeat the same assertion as in Lemma 1. Moreover, in the partition function form game situations, we do not have to consider the case of $a \neq a^*$ under π_N. Under coalition structure π_i, an action profile is fixed by the Nash equilibrium $a^{**}(\pi_i)$ in Definition 3. Then, we can prove the following lemma.

lemma 3' *In every pure strategy MPE at state $(\pi_i; a, t)$, every player ℓ, $\ell = 1, 2, 3$, proposes at the first round the solution of the maximization problem:*

$$\begin{aligned}
&\max_{(\hat{a}, \hat{t}, S)} u_\ell(\hat{a}) + \hat{t}_\ell + \delta\phi_\ell(\hat{\pi}; \hat{a}, \hat{t}) \\
&\text{subject to } u_m(\hat{a}) + \hat{t}_m + \delta\phi_m(\hat{\pi}; \hat{a}, \hat{t}) \geq x_m(\pi_i; a, t), \text{for } m \in S, m \neq \ell, \\
&\qquad \hat{a} = (\hat{a}_S, \hat{a}_{-S}), \text{where } \hat{a}_S \in A_S, \hat{a}_{-S} \in r_{-S}(\hat{a}_S), \text{ and } \hat{t} \in T_S, \\
&\qquad S \in \Psi_\ell(\pi_i), \hat{\pi} \in \{\pi_i, \pi_N\}.
\end{aligned}$$

Moreover, the proposal is accepted. The equilibrium proposal of player ℓ is given as follows: $\hat{a} = a^*$, $S = N$ and $\hat{t}$ such that $\hat{t}_\ell = -\hat{t}_m - \hat{t}_n$, $\hat{t}_m$ and $\hat{t}_n$ satisfying

$$\frac{1}{1-\delta}(u_m(a^*) + \hat{t}_m) = x_m(\pi_i; a, t), \text{ and } \frac{1}{1-\delta}(u_n(a^*) + \hat{t}_n) = x_n(\pi_i; a, t).$$

It is suffice to apply the same proof in Lemma 3 to the partition function form game situations. Every player proposes the grand coalition and a^* under coalition structure π_i in the partition function form game situations. Moreover, we can provide the following lemma corresponding to Lemma 4 in the initial state $(\pi_0; a^0, t^0)$.

Lemma 4'. *In every pure strategy MPE σ at the initial state $(\pi_0; a^0, t^0)$ in the partition function form* *game situation, every player i proposes either the solution of the maximization problem:*

$$\begin{aligned}&\max_{\hat{\pi}\in\{\pi_j,\pi_k\},\hat{t}} u_i(a^{**}(\hat{\pi}))+\hat{t}_i+\delta\phi_i(\hat{\pi};a^{**}(\hat{\pi}),\hat{t}),\\ &\text{subject to } u_\ell(a^{**}(\hat{\pi}))+\hat{t}_\ell+\delta\phi_i(\hat{\pi};a^{**}(\hat{\pi}),\hat{t}),\\ &\text{where } \ell=k \text{ if } \hat{\pi}=\pi_j,\ \ \ell=j \text{ if } \hat{\pi}=\pi_k, \quad \text{and } \hat{t}_i+\hat{t}_\ell=0,\end{aligned}$$

or the status quo. If the solution of the problem is chosen by player i, the proposal is accepted in σ, and if the status quo is chosen, the delay of agreements happens.

We omit the proof because we can apply the same procedure as Lemma 4, except the feasible action profile is restricted to $a^{**}(\pi_j)$ under π_j or $a^{**}(\pi_k)$ under π_k. In order to claim Proposition 2, we have to provide a condition which contradicts the statement that "a unacceptable proposal is the optimal strategy for every player in the initial state." This statement can be considered as the induction hypothesis. If every player proposes a unacceptable proposal (or the status quo) and, then, the economy permanently remains at the initial state, player i gets the expected discounted payoff of $(1/(1-\delta))u_i(a^0)$, $i=1,2,3$. If it is satisfied for coalition $\{i,j\}$ that

$$\begin{aligned}&u_i(a^{**}(\pi_k))+\delta\phi_i(\pi_k;a^{**}(\pi_k),t)+u_j(a^{**}(\pi_k))+\delta\phi_j(\pi_k;a^{**}(\pi_k),t)\\ &>\frac{1}{1-\delta}u_i(a^0)+\frac{1}{1-\delta}u_j(a^0),\end{aligned}\tag{19}$$

then both player i and j propose coalition $\{i,j\}$ in the coalition formation phase and the proposal is accepted. Thus, there exists some player to make an acceptable proposal. By Lemma 1' and 3', we have

$$\begin{aligned}&\phi_i(\pi_k;a^{**}(\pi_k),t)\\ &\quad=\frac{1}{1-\delta}(u_i(a^{**}(\pi_k))+t_i)+\frac{1}{3}\frac{1}{1-\delta}\left[\sum_{\ell=1}^{3}u_\ell(a^*)-\sum_{\ell=1}^{3}u_\ell(a^{**}(\pi_k))\right],\end{aligned}\tag{20}$$

$$\begin{aligned}&\phi_j(\pi_k;a^{**}(\pi_k),t)\\ &\quad=\frac{1}{1-\delta}(u_j(a^{**}(\pi_k))+t_j)+\frac{1}{3}\frac{1}{1-\delta}\left[\sum_{\ell=1}^{3}u_\ell(a^*)-\sum_{\ell=1}^{3}u_\ell(a^{**}(\pi_k))\right].\end{aligned}\tag{21}$$

Substituting (20), (21) to (19) and taking into account of $t_i+t_j=0$, we can obtain the condition (6) in Proposition 2.

References

[1] Agion, P., Antras, P. and Helpman, E. (2007), "Negotiating Free Trade," *Journal of International Economics* **73**, 1-30.

[2] Aumann, R. (1961), "The Core of a Cooperative Game without Side Payments," *Transactions of the American Mathematical Society* **98**, 539-552.

[3] Aumann, R. (1967), "A Survey of Cooperative Game without Side Payments," M. Shubik, ed., *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, Princeton University Press, Princeton, 3-27.

[4] Bloch, F. (1996), "Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division," *Games and Economic Behavior* **14**, 90-123.

[5] Bloch, F. and Gomes, A. (2006), "Contracting with Externalities and Outside Options," *Journal of Economic Theory* **127**, 172-201.

[6] Currarini, S. (2007), "Group Stability of Hierarchies in Games with Spillovers," *Mathematical Social Sciences* **54**, 187-202.

[7] Coase, R. (1960), "The Problem of Social Cost," *Journal of Law and Economics* **3**, 1-44.

[8] Deneckere, R. and Davidson, C. (1983), "Incentives to Form Coalitions with Bertrand Competition," *Rand Journal of Economics* **16**, 473-486.

[9] Diamantoudi, E. and Xue, L. (2007), "Coalitions, Agreements and Efficiency," *Journal of Economic Theory*, forthcoming.

[10] Funaki, Y. and Yamato, T. (1999), "The Core of an Economy with a Common Pool Resource: A Partition Function Form Approach, " *International Journal of Game Theory* **28**, 157-171.

[11] Furusawa, T. and Konishi, H. (2007), "Free Trade Networks," *Journal of International Economics* **72**, 310-335.

[12] Gomes, A. (2005), "Multilateral Contracting with Externalities," *Econometrica* **73**, 1329-1350.

[13] Gomes, A. and Jehiel, P. (2005), "Dynamic Processes of Social and Economic Interactions: On the persistence of inefficiencies," *Journal of Political Economy* **113**, 626-667.

[14] Goyal, S. and Joshi, S. (2006), "Bilateralism and Free Trade," *International Economic Review* **47**, 749-778.

[15] Hafalir, I. (2007), "Efficiency in Coalition Games with Externalities," *Games and Economic Behavior* **61**, 241-258.

[16] Hyndman, K. and Ray, D. (2007), "Coalition Formation with Binding Agreements," *Review of Economic Studies* **74**, 1125-1147.

[17] Ichiishi, T. (1981), "A Social Coalitional Equilibrium Existence Lemma," *Econometrica* **49**, 369-377.

[18] Jackson, M. and Wolinsky, A. (1996), "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory* **71**, 44-74.

[19] Kamien, M., Muller, E. and Zang, I. (1992), "Research Joint Ventures and R&D Cartels," *American Economic Review* **82**, 1293-1306.

[20] Macho-Stadler, I., Perez-Castrillo, D. and Porteiro, N. (2006), "Sequential Formation of Coalitions through Bilateral Agreements in a Cournot Setting," *International Journal of Game Theory* **34**, 207-228.

[21] Maskin, E. (2003), "Bargaining, Coalitions, and Externalities," mimeo, Institute for Advanced Study.

[22] Okada, A. (1996), "A Noncooperative Coalitional Bargaining Game with Random Proposers," *Games and Economic Behavior* **16**, 97-108.

[23] Okada, A. (2000), "Efficiency Principle in Non-Cooperative Coalitional Bargaining," *Japanese Economic Review* **51**, 34-50.

[24] Ray, D. (2008), *A Game-Theoretic Perspective on Coalition Formation*, Oxford University Press.

[25] Ray, D. and Vohra, R. (1999), "A Theory of Endogenous Coalition Structures," *Games and Economic Behavior* **26**, 286-336.

[26] Ray, D. and Vohra, R. (2001), "Coalitional Power and Public Goods," *Journal of Political Economy* **109**, 1335-1384.

[27] Segal, I. (1999), "Contracting with Externalities," *Quarterly Journal of Economics* **114**, 337-388.

[28] Seidmann, D. (2005), "Preferential Trading Arrangements as Strategic Positioning," mimeo, Nottingham University.

[29] Seidmann, D. and Winter, E. (1998), "A Theory of Gradual Coalition Formation," *Review of Economic Studies* **65**, 793-815.

[30] Shapley, L. and Shubik, M. (1969), "On the Market Games," *Journal of Economic Theory* **1**, 9-25.

[31] Yi, S-S. (1996), "Endogenous Formation of Customs Unions under Imperfect Competition: Open regionalism is good, " *Journal of International Economics* **41**, 153-177.

[32] Yi, S-S. (1997), "Stable Coalition Structures with Externalities," *Games and Economic Behavior* **20**, 201-237.

[33] Yi, S-S. (1998), "Endogenous Formation of Joint Ventures with Efficiency Gain," *Rand Journal of Economics* **29**, 610-631.

[34] Yi, S-S. and Shin, H. (2000), "Endogenous Formation of Research Coalitions with Spillovers," *International Journal of Industrial Organization* **18**, 229-256.