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**Abstract**

This note provides a simple example in which the delay of agreement happens even in a noncooperative coalitional bargaining model with random proposers for a non-superadditive game.

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# 1 Introduction

Chatterjee et al. (1993) show that the delay of agreement may occur in a stationary equilibrium for $n$-person coalitional bargaining model, in contrast to the Rubinstein (1982) two-person alternating-offer model. In their model, a proposer is determined in a fixed order over the players, and the first rejecter becomes the next proposer. Okada (1996) points out that the delay of agreement is caused by their fixed protocol of bargaining. He presents the bargaining model in which the proposer is randomly selected at every round, and shows that no delay of agreement occurs in the random-proposer model. In his paper, an $n$-person coalitional game $(N, v)$ is assumed to be superadditive. Superadditivity of a game means that the coarsest coalition structure consisting of the grand coalition can generates the maximum of total feasible payoff of all players in $N$. Some economic environments, however, could not be described by a class of superadditive game. Geusnerie and Oddou (1979, 1981) and Greenberg and Weber (1986) have clearly shown that the characteristic form game is not necessary superadditive where the local public good is financed through a proportional income tax or a poll tax. As shown in Mutuswami, Perez-Castrillo and Wettstein (2004), the game of the local public goods economy with congestion effects is not superadditive even if the tax system to finance the public good is flexible enough to adjust contributions for each individual[1]. Congestion is one of important factors to generate non-superadditivity in a local public goods economy. In addition, the cooperative game associating to the network formation with costs for forming links can be nonsuperadditive, as shown in van den Nouweland (2005). In this note, we give an example in which the delay of agreement occurs even in the random-proposer model when a coalitional form game is not superadditive.

# 2 Coalitional Bargaining

We consider a noncooperative coalitional bargaining model with random proposers by Okada (1996). A bargaining situation is described by an $n$-person coalitional form game $(N, v)$ with transferable utility. $N = \{1, 2, \ldots, n\}$ is the set of players and $v : \Sigma \to \mathbb{R}$ is a characteristic function, where $\Sigma$ denotes the set of all coalitions. Here, we focus on the case in which the characteristic

[1]Mutuswami et al. (2004) does not refer to congestion effects by the members of coalition. They provide the model in which both the utility function and the cost function of the public good depend on the coalition to which the players belongs. These dependences can be interpreted as congestion effects.

function $v$ is not *superadditive*. Thus, there exists two disjoint coalitions $S$ and $T$ in $\Sigma$ such that $v(S)+v(T) > v(S \cup T)$. A payoff vector for a coalition $S$ is denoted by $x^S = (x_i^S)_{i \in S}$. It is called feasible if $\sum_{i \in S} x_i^S \leq v(S)$. $X_+^S$ denotes the set of all feasible payoff vectors with nonnegative components.

A noncooperative bargaining game is as follows. At every round $t = 1, 2, \ldots,$ one player is selected as proposer with equal probability among all active players $N^t$. The bargaining starts with all players, i.e., $N^1 = N$. The selected player $i$ makes a proposal of a coalition $S$ with $i \in S \subset N^t$ and of a payoff vector $y^S \in X_+^S$. All other players in $S$ respond sequentially by accepting or rejecting the proposal. If every player in $S \backslash \{i\}$ accepts the proposal $(y^S, S)$, it is agreed and enforced. In this case, the coalition $S$ is formed and every player $j \in S$ gets the payoff $y_j^S$. Then, the remaining players outside $S$ can continue bargaining at the next round. Thus, $N^{t+1} = N^t \backslash S$. If some player in $S$ rejects the proposal, bargaining proceeds to the next round with same active players. The process continues until there is no subset $S$ of the remaining players with $v(S) > 0$. When a proposal $(y^S, S)$ is agreed upon at round $t$, the payoff of every member $i \in S$ is $\delta^{t-1} y_i^S$, where $\delta$ is a discount factor. For players who reach no agreement, their payoffs are assumed to be zero.

We adopt a *stationary subgame perfect equilibrium* (SSPE) as a solution concept. The concept of an SSPE is used in almost all literatures of multilateral bargaining model (see Chatterjee et al., 1993, Okada, 1996).

# 3 Example of Delay

We consider a four-person game; $N = \{1, 2, 3, 4\}$,

$$\begin{aligned} &v(\{1,2,3,4\}) = 120, \\ &v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = 50, \\ &v(\{i,j\}) = 100, \quad \text{for } i,j = 2,3,4, \end{aligned}$$

and other coalitions are infeasible. Because $v(\{1,k\}) + v(\{i,j\}) > v(\{1,2,3,4\})$ for $i,j,k = 2,3,4$, the above game is not superadditive. We assume that the discount factor is almost one. Consider the following strategies for players. If the set of active players is $N = \{1,2,3,4\}$, player 1 proposes $(\{1,2\}, (50, 0))$, player 2 does $(\{2,3\}, (100 - 125/3, 125/3))$, player 3 does $(\{3,4\}, (100 - 125/3, 125/3))$, and player 4 does $(\{2,4\}, (125/3, 100 - 125/3))$. The response rule for players is as follow. Player 1 accepts a proposal $(y, T)$ if and only if $y_1 \geq 25$ for all $1 \in T$. Player 2, 3, 4 accepts any proposal if and only if $y_j \geq 125/3$ for $j = 2, 3, 4$ and $j \in T$.

When the set of active players is $\{1, i\}$, $i = 2, 3, 4$, a player selected as a proposer proposes $(\{1, i\}, (25, 25))$, and accepts any proposal if he is offered a payoff equal to or greater than 25. When the set of active players is $\{i, j\}$, $i, j = 2, 3, 4$, every player proposes $(\{i, j\}, (50, 50))$ and accepts any proposal if his payoff is equal to or greater than 50.

When the above strategy is used, the expected payoff of player 1 is 25 and that of player 2, 3, 4 is 125/3. It is easy to see that the strategy constructed is a SSPE in the bargaining game model. Let us consider a subgame in which only two players are still active. In two-person bargaining with random proposers, the strategy such that a proposer demands a half of pie and a responder accepts any proposal if his payoff offered is equal to or greater than a half of pie is optimal when a discount factor is almost one. Therefore, the above strategy composes a subgame perfect equilibrium point in the subgame when the set of remaining players is $\{i, j\}$, $i, j = 1, 2, 3, 4$.

Next consider a bargaining game in which four players are active. We can check the optimality of the response rule of every player. If player 1 rejects an offer in the four-person bargaining, negotiations go to the next round, and his expected payoff will be 25. Thus, it is optimal for him to accept any offer in the four-person bargaining if he get at least 25. Similarly, if player $i$, $i = 2, 3, 4$ rejects an offer, he can gain the expected payoff 125/3. It is optimal for him to accept the offer in the four-person bargaining if he can get at least 125/3.

Let us check the optimality of every player's proposal in the above strategy. Given the response rule of other players, player 2 can obtain (100−125/3) by proposing coalition $\{2, 3\}$, and also (100−125/3) by proposing $\{2, 4\}$. If he proposes a four-person coalition $\{1, 2, 3, 4\}$, he obtain only 35/3(=(120−25−250/3)). In addition, we can get 25 by proposing coalition $\{1, 2\}$. Thus, it is optimal for him to propose coalition $\{2, 3\}$ with demanding (100−125/3). We can prove the optimality of the proposals of player 3, 4 in the same way.

If player 1 proposes $(\{1, 2\}, (50, 0))$ defined above, then player 2 rejects the proposal, and negotiations go to the next round. Thus, player 1 gets the expected payoff 25. In order to form a four-person coalition $\{1, 2, 3, 4\}$, player 1 has to guarantee the continuation payoff 125/3 for player 2, 3, 4 respectively. Because $v(\{1, 2, 3, 4\}) = 100$, player 1 could not make a feasible proposal for coalition $\{1, 2, 3, 4\}$. Moreover, he obtains at most (50−125/3) ($< 25$) by proposing an acceptable offer for coalition $\{1, i\}$, $i = 2, 3, 4$. Thus, it is optimal for player 1 to make his proposal rejected at round 1, for example, the proposal $(\{1, 2\}, (50, 0))$. As results, delay of agreement occurs when player 1 is selected as a proposer at round 1.

If a game is superadditive, every player can get at least the expected payoff by proposing the grand coalition when he is a proposer. This fact leads to no delay of agreement in a noncooperative coalitional bargaining model with random proposers. However, when a game is not superadditive, forming the grand coalition does not ensure the expected payoff for a proposer. In the example presented, the sum of expected payoffs of all players (25+125/3+125/3+125/3) (=150) could not be feasible in the grand coalition; $150 > v(\{1, 2, 3, 4\}) = 100$. Each player benefits from forming a smaller coalition than the full coalition by avoiding congestion. Moreover, it becomes important for each player to form a coalition with whom and when to form. In our example, if player $i$, $i = 2, 3, 4$ and player 1 remain at bargaining after player $j, k = 2, 3, 4$, $j, k \neq i$ form a two-person coalition, then, player $i$ can obtain only 25. On the other hand, player $j$ and $k$ can get at least 125/3. Thus, player $i$, $i = 2, 3, 4$ has an incentive to form a two-person coalition $\{i, j\}$ with $j = 2, 3, 4$ at the first round. Player 1 is in a weaker position in the bargaining than player 2, 3, 4. Player 1 cannot benefit in the bargaining with the three strong players at round 1. Therefore, he optimally waits for a two-person coalition of strong players to be formed. This is an unknown strategic aspect of coalition formation. Nonsuperadditivity, for example caused by congestion, may induce the delay of agreement even in the random-proposer model.

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