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Can a minimum wage hike decrease an unemployment rate?

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abstract

There are lots of studies on the minimum wage policy that is very controversial. However, few studies focuses on the minimum wage policy under demand shortage in the goods market that often occurs in reality. This paper argues how an increase in minimum wage affects employment, consumption, and social welfare with dynamic general equilibrium model. The study demonstrates that a minimum wage hike has positive effects on an employment rate, aggregate consumption, and welfare under a demand shortage economy whereas does not under a non-demand shortage. This implies that the minimum wage policy can be considered as one of the option of the economic recovery policy without any government spending although it increases the natural unemployment rate.

KEYWORDS: Unemployment, Deflation, Stagnation, Minimum wage, Demand shortage, Dynamic general equilibrium model
JEL Classification Codes: E24 E31 J38

1 Introduction

The "textbook" model of the competitive labor market states that a decline in wages increases the employment, firm's profits and welfare and thus vitalizes the economy. However, the real wage in Japan has shown a mild decrease tendency since the late 1990s as in Fig.1, and the Japanse economy has been still stagnating. Why does not the decrease in wages stimulate an economy as the orthodox theory shows? This paper argues that the impact of minimum wage policy on the economy differs in response to the economic situation, and I show that when an economy faces a demand shortage, an increase in wages improves the employment rate, aggregate consumption, and social welfare. On the

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other hand, in an economy that does not face demand shoratage, the increase in wages worsens the employment rate, aggregate consumption, and social welfare. Trade unions often insist on wage hikes during the labor-management negotiations, arguing that wage hikes will stimulate the consumption and aggregate demand. This paper supports these opinions in the context of an sluggish economy but not in the context of an booming economy.

[Insert Fig.1]

I develop a simple extension of Ono's (2001) dynamic general equilibrium model with two different types of jobs. In the model economy, a single good can be produced by two labor inputs. Firms paid an efficiency wage and a minimum wage for each job. This assumption gives rise to a positive link between efficiency wage and minimum wages that can be used to analyze the effect of a wage hike (caused by a minimum wage hike) on the economy. This wage setting enalbles a tractable analysis because the minimum wage hike leaves the relative wage of each job unchanged, which eliminates the effect of substitution of labor demand for each job.[1] Therefore, the model can focus on the other effects of the minimum wage hike such as on inflation and the budget constraint of households that is unnoticed earlier.

Households have utility from consumption and real balances of money. Assumption of the marginal utility of money brings about two different equilibrium: a demand-shortage and a non-demand-shortage economy as Ono (2001) showed. If the marginal utility of money is insatible, the households accumulate money more than enough, and hence, the aggregate comsumption level falls short of aggregate output level, that is, the demand-shortage equilibrium comes out. If the marginal utility of money is satible, the non-demand-shortage equilibrium shows up. In the analysis, contrasting a demand-shortage and non-demand-shortage economy sheds new light on the function of a minimum wage.

In a demand-shortage economy, the minimum wage hike can prominently increase aggregate consumption, decrease the unemployment rate, and improve social welfare. The reason for this result is attributable primarily to a firm's labor demand function. When the firm faces the demand-shortage constraint in a demand-shorage economy, a equilibrium of underemployment arises in which the marginal product of labor is higher than the wage. Hence higher aggregate demand induces the firms to increase their labor demand and to decrease the underemployment as Barro and Grossman (1971) showed. At the same time, an increase in the minimum wage narrows the disequilibrium gap between demand and supply caused by the stimulation of consumption and then eases deflation. This analysis also provides a policy implication for governments concerned about budget

[1] Cahuc and Michel (1996) state that the minimum wage hike increases the relative wage of unskilled jobs and induces firms to substitute other jobs including skilled jobs for unskilled jobs.

deficit–that is, a minimum wage hike can raise the aggregate demand without an increase in government spending.[2]

On the other hand, in a non-demand-shortage economy, a minimum wage hike decreases the employment and worsens the social welfare in analogy with the orthodox theory.

A number of studies are related to this work. As stated above, I build on Ono's (2001) dynamic general equilibrium model with both non-demand-shortage and demand-shortage economies; this model does not discuss the impact of minimum wage but shows an exact reason for the occurrence of a demand-shortage economy.

The minimum wage policy is controversial, and lots of empirical studies analyze the minimum wage effects on the employment.[3] Several theoretical studies argue the positive function of minimum wage as opposed to the competitive model. The welfare-enhancing minimum wage policy can be obtained by the intensifying capital accumulation. Cahuc and Michel (1996) and Fanti and Gori (2011) consider a growth model in which minimum wage hikes can improve welfare, but for reasons that are different from those we consider here.[4] In their model, a minimum wage hike increases savings and capital accumulation at the cost of increasing unemployment. It then improves economic growth and welfare under generous unemployment benefits and the positive externality of human capital accumulation that stem from the substituting skilled labor for binding minimum-wage low-skilled labor. In a context of search model Flinn (2006) show that minimum wage improves unemployment inefficiency and may increase employment considering the size of the searching participants in response to the minimum wage.[5] Furthermore, in the monopsony model as is well known, a minimum wage hike can increase the employment and improve the welfare.[6] Moreover, Revitzer and Taylor (1995) builds on the sharking

[2]Inflation in the eurozone has fallen to 0.7 % in October 2013 – its lowest level since January 2010. ECB has expressed concerned about further diminishing price pressures, and has lowered the interest rate to 0.25 %. It is also worth considering the minimum wage policy to stumulate the economic activity and to alleviate the diminishing price pressures.

[3]Lately, Newmark, Salas and Wascher (2013) conducts a controversy with Allegretto, Dube, and Reich (2011), and Dube, Lester, and Reich (2010).

[4]Some studies focus on the relation economic growth and minimum wage. Irmen and Wigger (2006) consider a two-country overrapping-generations model with capital mobility and endogenous growth, which shows the condition that minimum wage increases the global economic growth. Meckel (2004) also considers an endogenous growth model comprising the sectors of final goods, intermediate goods, and R&D. This model shows that higher minimum wages for unskilled labor leads to increased growth and unskilled unemployment, while possibly reducing unemployment of skilled labor. Tamai (2009) considers a median voter model of heterogeneous households with endogenous growth that determines the minimum wage by voting. He finds that high inequality has a positive effect on the minimum wage but generates a non-monotonic relation between inequality and economic growth.

[5]Acemoglu (2001) constructs a search model in which high and low-wage jobs coexist in response to capital intensity of each industry and demonstrates that introducing a minimum wage shifts the composition of employment toward high-wage jobs, increases average labor productivity, and may improve welfare.

[6]Bhaskar and To (1999) constructs the monopsonistic competition, where a large number of employers compete for workers, and are able to freely enter or exit. A rise in minimum wage raises employment per firm but causes firm's exit due to the decline of their profit. If the labor market is sufficiently distorted, the rise in minimum wage raises aggregate employment and welfare.

model of Shapiro and Stiglitz (1984) and shows that the increase in minimum wage may raise the employment rate. None of these studies focuses on the minimum wage policy under demand shortage in the goods market that often occurs in reality.

The rest of the paper is organized as follows. Section 2 explains the basic setting of the model. The effects of a minimum wage are analyzed in a non-demand-shortage economy in section 3 and in a demand-shortage ecnomy in section 4. Section 5 concluds the paper.

2 The model

2.1 Firms

Consider an economy in which firms produce final goods that are sold in a cometitive market with two labor inputs. A feature of one of two labor (job) is that workers' effort increase the output, and hence the firm pays the efficiency wage for this labor. This type of labor is called a high-wage job. The other labor feature is that the worker's effort cannot increase the output, and hence the firm pays the minimum wage that is more than the competitive wage level for this labor because I assume that the minimum wage is regulated in the economy. This is called a low-wage job.[7]

The production function for each firm is concave and is given by

$$y = (en_1)^a\, n_2^b, \quad 0 < a, b < 1, \tag{1}$$

where y denotes the amounts of output produced and n_1 and n_2 stand for the number of high-wage and low-wage job employees, respectively. e indicates productivity affected by the worker's effort, and its functional form is

$$e = \left(\frac{w_1 - x}{x}\right)^{\theta}, \quad 0 < \theta < 1, \tag{2}$$

where x is the reference point that equals reservation wage (See below in detail). Equation (2) shows that an increase in wage margin from the reservation wage raises productivity. The representative firm chooses a level of real wages that minimize costs per unit of the effective labor input, w_1/e. The optimal wage and effort level are

$$\bar{w}_1 = \frac{x}{1-\theta}, \tag{3}$$

$$\bar{e} = \left(\frac{\theta}{1-\theta}\right)^{\theta}. \tag{4}$$

[7] I assume that the low-wage job worker does not shirk, because the firm utilizes monitoring technology.

The firm is a price taker and sells the final goods at a price P. Thus, the representative firm faces the following profit maximization problem:

$$\max_{n_1,n_2} Py - W_1 n_1 - W_2 n_2$$
$$s.t. \quad y = (\bar{e}n_1)^a\, n_2^b, \quad y = y^d (under \quad demand \quad shortage \quad regime),$$

where W_1 and W_2 are nominal wages of each type of worker, and y^d is the aggregate demand. When the firm can sell all the output produced without considering demand shortage, the optimal conditions are

$$\bar{e}a\,(\bar{e}n_1)^{a-1}\, n_2^b = w_1, \tag{5}$$
$$b\,(\bar{e}n_1)^a\, n_2^{b-1} = w_2. \tag{6}$$

I refer to this economy supply-side (non-demand-shortage) regime because the output is determined not by demand-side factors but by supply-side factors.

On the other hand, when the firm faces the demand shortage, the optimal conditions are[8]

$$(\bar{e}n_1)^a\, n_2^b = y^d, \tag{7}$$
$$\frac{an_2}{bn_1} = \frac{w_1}{w_2}. \tag{8}$$

I refer to this economy as a demand-shortage regime. In fact, the marginal product of employment n_2 is higher than w_2 because the employment n_2 determined under the demand-shortage constraint in (7) and (8) is lower than the employment n_2 determined under the no demand-shortage constraint in (5) and (6).

2.2 Households

Infinitely lived households have a utility function of the form

$$U = \int_0^\infty e^{-\rho t}\,[u(c) + v(m)]\,dt, \tag{9}$$

[8] As is shown by Ono (2001), the demand shortage arises not due to price rigidity but due to households' preference. The firms know that the demand shortage cannot be eliminated in spite of price adjustments, and hence, firms maxmize their profits given the constraint of demand shortage. The optimal condition of the problem can be written by the Lagrangian multiplier method as follows:

$$L = P\,(\bar{e}n_1)^a\, n_2^b - W_1 n_1 - W_2 n_2 + \lambda((\bar{e}n_1)^a\, n_2^b - y^d)$$

The optimal conditions are

$$(1+\lambda)\bar{e}a\,(\bar{e}n_1)^{a-1}\, n_2^b = w_1,$$
$$(1+\lambda)b\,(\bar{e}n_1)^a\, n_2^{b-1} = w_2,$$
$$(\bar{e}n_1)^a\, n_2^b - y^d = 0.$$

where ρ is a constant rate of time preference, and $u(c)$ and $v(m)$ are a continuous quasi-concave instantaneous utilities of real consumption c and real money balances m, respectively. I abbreviate the time notation of each variable to simplify exposition. The households provide one unit of labor inelastically. I assume the population size is equal to 1. The households are ex ante identical and the allocation of their labor to high-wage or low-wage jobs is done through a lottery. The households are then divided into two types by their employment status, with each type having different budget constraints. One engages in the high-wage job that receives the efficiency wage, and the other engages in the low-wage job that receives the minimum wage.

Each household choose the optimal consumption and the real money balances to maximize U, subject to the following flow budget constraint:

$$\dot{m}_1 = w_1 + \frac{q}{n_1} - z_1 - c_1 - \pi m_1, \quad (i=1), \tag{10}$$

$$\dot{m}_2 = w_2 - z_2 - c_2 - \pi m_2, \quad (i=2). \tag{11}$$

Each household's variables are denoted by suffix $i = 1$ if he/she has the high-wage job, and $i = 2$ for the low-wage job.[9] $w_i \equiv W_i/P$ is the real wage rate, $z_i (\geq 0)$ is a lump-sum tax. I assume that a firm's real profit, $q \equiv f(n_1, n_2) - w_1 n_1 - w_2 n_2$, is equally distributed to the households of high-wage job.[10] π is the inflation rate. Then, the first-order optimal conditions are

$$\eta_{c_i} \frac{\dot{c}_i}{c_i} = \frac{v'(m_i)}{u'(c_i)} - \rho - \pi, \quad (i = 1, 2), \tag{12}$$

where $\eta_{c_i} \equiv \frac{-u''(c_i)c_i}{u'(c_i)} > 0$. The transversality conditions are

$$\lim_{t \to \infty} \lambda(t) m_i(t) e^{-\rho t} = 0, \quad (i = 1, 2), \tag{13}$$

where $\lambda(t)$ is a costate variable of m_i.

At any point in time, the money market is in equilibrium.

$$m^s = n_1 m_1 + n_2 m_2, \tag{14}$$

where m^s indicates the real money stock. The percentage change in the money stock depends on the government's money expansion rate, $\mu \equiv \frac{\dot{M}^s}{M^s}$, and the inflation rate, π.

$$\frac{\dot{m}^s}{m^s} = \mu - \pi. \tag{15}$$

[9] The behavior of unemployed people is not considered in the model. That is, the model assumes implicitly that unemployed people are parasites on their friends or relations who have a job.

[10] This assumption is for the simplicity. Meanwhile, it is possible to build in the stock market, in which case the firm's profits are wholly distributed as dividends. This setting makes the model too tangled because it needs to endogenize a rate of profit return or stock price.

Gorvernment spending g is financed by monetary expansion and households' taxation. Therefore, the government's budget constraint is

$$g = m^s \mu + n_1 z_1 + n_2 z_2, \tag{16}$$

where $m^s \mu = \frac{M^s}{P} \frac{\dot{M}^s}{M^s}$.

The aggregate demand consists of the consumption of households and the gorvernment spending.

$$y^d = c_1 n_1 + c_2 n_2 + g. \tag{17}$$

2.3 Wage determination

The firm's setting of efficiency wage depends on the reservation wage x, which equals to the worker's expectation wage when he/she loses the present job.[11]

$$x = n_1 w_1 + n_2 w_2. \tag{18}$$

Substituting (18) into (3) yields

$$w_1 = \frac{n_2 w_2}{1 - \theta - n_1}. \tag{19}$$

Equation (19) predicts that the circumstances of the labor market affect the wage of high-wage job workers, that is, a minimum wage hike and improvements in each employment rate raise the wage of the high-wage job workers. I assume $\theta < 1 - n_1$ to secure the existence of a solution.

3 Minimum wage effects in a supply-side regime

Using (19), (5) and (6), the employment rates for each type of labor in the supply-side regime economy are

$$n_1 = \frac{a(1-\theta)}{a+b} \equiv n_1^s, \tag{20}$$

$$n_2 = \left(\frac{b}{w_2}\right)^{\frac{1}{1-b}} \left(\frac{\bar{e}a(1-\theta)}{a+b}\right)^{\frac{a}{1-b}} \equiv n_2^s(w_2), \quad \frac{dn_2^s}{dw_2} = n_2^{s\prime}(w_2) < 0. \tag{21}$$

The increase in minimum wage reduces the employment rate n_2 but does not affect the employment rate n_1,[12] In the supply-side regime, employment is determined by only the

[11] Falk, Kehr, and Zehnder (2006) shows minimum wages have significant effects on reservation wages with a laboratory experiment.

[12] This is because this model assumes the Cobb-Dougulas production function. $\frac{\partial n_1}{\partial w_2} > 0$ when $\frac{f_{22} n_2}{f_2} - \frac{f_{12} n_2}{f_1} + 1 > 0$ with concave non-homothetic production function $f(n_1^s, n_2^s(w_2))$ or elasticity of substitution is more than 1 with CES production function, where $f_j \equiv \frac{\partial f(n_1^s, n_2^s)}{\partial n_j}$, $(j = 1, 2)$ and $f_{jk} = \frac{\partial f_j(n_1^s, n_2^s)}{\partial n_k}$, $(k = 1, 2)$. I do not consider this case because this paper focuses on the other effects of minimum wage.

labor market variable. The output level becomes $y^s(w_2) \equiv (\bar{e}n_1^s)^a \left(n_2^s(w_2)\right)^b$ and the real profit level becomes $q^s(w_2) \equiv y^s(w_2) - \frac{n_1^s n_2^s(w_2) w_2}{1-\theta-n_1^s} - n_2^s(w_2)w_2$.

In analogy with Ono(2001), the gap between aggregate supply and demand is adjusted by the price

$$\pi = \alpha \left[\frac{y^d}{y^s(w_2)} - 1 \right], \quad \alpha > 0, \tag{22}$$

where π denotes the inflation rate and α stands for the adjustment speed of the price. In the supply-side regime, excess demand (supply) pushes up (down) the inflation rate and it becomes plugged in the steady state, in contrast to the demand-shortage regime below.

Combining (17), (20), and (21) with (22), the inflation rate in the supply-side regime becomes

$$\pi^s = \alpha \left[\frac{c_1 n_1^s + c_2 n_2^s(w_2) + g}{y^s(w_2)} - 1 \right],$$
$$\pi_{c_i}^s \equiv \frac{\partial \pi^s}{\partial c_i} = \frac{\alpha n_i^s}{y^s} > 0, \quad \pi_{w_2}^s \equiv \frac{\partial \pi^s}{\partial w_2} = Sign \left[(c_2 - w_2) \frac{n_2^{s'}(w_2) w_2}{y^s} \right] > 0, \tag{23}$$

where $c_2 - w_2 < 0$ in the equilibrium. Moreover, substituting (23) into (10), (11), and (12) yields

$$\dot{c}_1 = \frac{c_1}{\eta_{c_1}} \left[\frac{v'(m_1)}{u'(c_1)} - \rho - \alpha \left(\frac{c_1 n_1^s + c_2 n_2^s(w_2) + g}{y^s(w_2)} - 1 \right) \right], \tag{24}$$

$$\dot{c}_2 = \frac{c_2}{\eta_{c_2}} \left[\frac{v'(m_2)}{u'(c_2)} - \rho - \alpha \left(\frac{c_1 n_1^s + c_2 n_2^s(w_2) + g}{y^s(w_2)} - 1 \right) \right], \tag{25}$$

$$\dot{m}_1 = w_1 + \frac{q^s(w_2)}{n_1} - z_1 - c_1 - \alpha \left(\frac{c_1 n_1^s + c_2 n_2^s(w_2) + g}{y^s(w_2)} - 1 \right) m_1, \tag{26}$$

$$\dot{m}_2 = w_2 - z_2 - c_2 - \alpha \left(\frac{c_1 n_1^s + c_2 n_2^s(w_2) + g}{y^s(w_2)} - 1 \right) m_2. \tag{27}$$

Equations (24)-(27) form an autonomous dynamic system with respect to c_1, c_2, m_1 and m_2. The saddle-path stability of the dynamics is shown in Appendix A.1. In the steady state, consumption and real money balances of each household become constant, the gap between aggregate supply and demand is plugged, that is, $\pi = 0$, and the employment rate of each job is still determined by (20) and (21). Using (24)-(27), the steady state equilibrium values are obtained (see Appendix A.2) as follows:

$$c_i^{s*} = c_i^s(w_2), \quad \frac{dc_1^{s*}}{dw_2} < 0, \quad \frac{dc_2^{s*}}{dw_2} = sign(1 + m_1^{s*}\pi_{c_1}^s + m_2^{s*}\pi_{c_1}^s \frac{n_2^s}{n_1^s} - m_2^{s*}\pi_{w_2}^s) \tag{28}$$

$$m_i^{s*} = m_i^s(w_2), \quad \frac{dm_1^{s*}}{dw_2} < 0, \quad \frac{dm_2^{s*}}{dw_2} = sign(1 + m_1^{s*}\pi_{c_1}^s + m_2^{s*}\pi_{c_1}^s \frac{n_2^s}{n_1^s} - m_2^{s*}\pi_{w_2}^s) \tag{29}$$

The increase in the minimum wage entails an overall wage rise, and which affects consumption and money holdings through each household's budget and the inflation rate. In the steady state equilibrium, the increase in the minimum wage reduces the consumption and money balances of high-wage job households c_1, m_1 and low-wage job employment n_2, but it can raise the c_2, m_2 when the effect of $\pi^s_{w_2}$ in (28) and (29) is small enough. On the whole, the minimum wage hike lowers the aggregate consumption level, that is, $\frac{d(c_1 n_1^s + c_2 n_2^s)}{dw_2} < 0$, because the negative effects of c_1 and n_2 dominates the effects of c_2.

In the steady state, social welfare V^s can be expressed as

$$\begin{aligned} V^s &= \int_0^\infty n_1^{s*}\left(u(c_1^{s*}) + v(m_1^{s*})\right)e^{-\rho t}dt + \int_0^\infty n_2^{s*}\left(u(c_2^{s*}) + v(m_2^{s*})\right)e^{-\rho t}dt \\ &= \frac{1}{\rho}\left[n_1^{s*}\left(u(c_1^{s*}) + v(m_1^{s*})\right) + n_2^{s*}\left(u(c_2^{s*}) + v(m_2^{s*})\right)\right]. \end{aligned} \tag{30}$$

Differentiating (30) with minimum wage w_2 through $c_i^{s*}, m_i^{s*}, (i = 1, 2), n_2^{s*}$, Proposition 1 is obtained (see Appendix A.3).

Proposition 1 *In a supply-side regime economy, a minimum wage hike reduces the social welfare if* $\varepsilon > \frac{-1}{\frac{dc_1^{s*}}{dw_2}n_1^s}\left[\left(\frac{dc_1^{s*}}{dw_2}n_1^s + \frac{dc_2^{s*}}{dw_2}n_2^s\right)\left(u'(c_2^{s*}) + \rho\frac{u''(c_2^{s*})}{v''(m_2^{s*})}\right) + \frac{dn_2^s}{dw_2}\left(u(c_2^{s*}) + v(m_2^{s*})\right)\right]$.

Where ε indicates a subtraction $u'(c_2^{s*}) + \rho\frac{u''(c_2^{s*})}{v''(m_2^{s*})}$ from $u'(c_1^{s*}) + \rho\frac{u''(c_1^{s*})}{v''(m_1^{s*})}$. If ε is a positive value or higher than a certain negative value as in Proposition 1, the ambiguous effects of minimum wage hike on consumption c_2^{s*} in (28) and money balances m_2^{s*} in (29) become lower than the decreasing effects on c_1^{s*} and m_1^{s*}. Therefore the minimum wage hike reduces the social welfare.

The unemployment rate are can be expressed as $u^s = 1 - n_1^s - n_2^s(w_2)$. In general equilibrium the employment rate are still determined by (20) and (21) and hence, we obtain Proposition 2:[13]

Proposition 2 *In a supply side regime economy, a minimum wage hike reduces the aggregate output and raises the unemployment rate.*

As Blanchard and Katz (1997) argues that the increase in the reservation wage shifts up the "supply wage relation" and raise the natural rate of unemployment rate,[14] Proposition 2 implies that the minimum wage hike raises the natural rate of unemployment.

4 Minimum wage effects in a demand-shortage regime

In the supply-side (non-demand-shortage) regime economy, I assume implicitly that as the households increase their real money balances, the marginal utility of money converges

[13] $\frac{\partial u^s}{\partial w_2} > 0$ when $\frac{f_{22}n_2}{f_2} - \frac{f_{12}n_2}{f_1} + 1 + \frac{f_{11}n1}{f_1} - \frac{f_{21}n_1}{f_2} - \frac{n_1}{1-\theta-n_1} > 0$ with a concave non-homothetic production function or elasticity of substitution σ with CES production function satisfies $\frac{2}{\sigma} > 1 - \frac{n_1}{1-\theta-n_1}$.

[14] Blanchard and Katz also argues that when the increase in the reservation wage is proportion to the productivity growth, the natural rate of unemployment rate remains constant.

to zero, that is, $\lim_{m_i \to \infty} v'(m_i) = 0, \quad (i = 1, 2)$. Here, the marginal utility of money is assumed to be insatiable along with Ono (2001).[15]

$$\lim_{m_i \to \infty} v'(m_i) = \beta > 0, \quad (i = 1, 2) \tag{31}$$

Assumption (31) invokes a demand shortage, even if the price adjusts a disequilibrium of demand and supply in the goods market. In addition, as the low-wage households cannot save too much money, their marginal utility of money is assumed to be

$$v'(m_2) > \beta, \quad \text{for all} \quad m_2(t). \tag{32}$$

In the demand-shortage regime economy, the employment rates n_1, n_2 are affected by the labor and goods markets. Combining (7), (8), (17), and (19) yields

$$n_1^d = \frac{a(1-\theta)}{a+b} \equiv n_1^d, \tag{33}$$

$$n_2^d = n_2^d(c_1, c_2), \quad n_{2c_1} = \frac{\partial n_2}{\partial c_i} = \frac{c_i n_i}{\frac{\partial((\bar{e}n_1)^a n_2^b)}{\partial n_2} - c_2} > 0, \tag{34}$$

where $\frac{\partial((\bar{e}n_1)^a n_2^b)}{\partial n_2} - c_2 > w_2 - c_2 = z_2 + \dot{m}_2 + \pi m_2 > 0$ because marginal product of n_2 is higher than w_2 owing to the demand-shortage.[16] Note that the employment rate of high-wage job n_1^d is constant as is equals n_1^s; whereas the employment rate of low-wage job n_2^d depends not on wage but on consumption of each household, and is significantly less than n_2^s because firms faces the aggregate demand constraint.

Substituting (33) and (34) into (22) yields

$$\pi^d(c_1, c_2, w_2) = \alpha \left[\frac{c_1 n_1^d + c_2 n_2^d(c_1, c_2) + g}{y^s(w_2)} - 1 \right], \quad \pi^d_{c_i} \equiv \frac{\partial \pi^d}{\partial c_i} > 0, \quad \pi^d_{w_2} \equiv \frac{\partial \pi^d}{\partial w_2} > 0. \tag{35}$$

π^d is expressed as a positive function of c_1, c_2, and w_2.

Considering assumptions (31) and (32), employment rates (33) and (34), and inflation rate (35), the dynamic system of the demand-shortage regime is

$$\dot{c_1} = \frac{c_1}{\eta_{c_1}} \left[\frac{\beta}{u'(c_1)} - \rho - \alpha \left(\frac{c_1 n_1^d + c_2 n_2^d(c_1, c_2) + g}{y^s(w_2)} - 1 \right) \right], \tag{36}$$

$$\dot{c_2} = \frac{c_2}{\eta_{c_2}} \left[\frac{v'(m_2)}{u'(c_2)} - \rho - \alpha \left(\frac{c_1 n_1^d + c_2 n_2^d(c_1, c_2) + g}{y^s(w_2)} - 1 \right) \right], \tag{37}$$

$$\dot{m}_2 = w_2 - z_2 - c_2 - \alpha \left(\frac{c_1 n_1^d + c_2 n_2^d(c_1, c_2) + g}{y^s(w_2)} - 1 \right) m_2. \tag{38}$$

[15] Ono, Ogawa and Yoshida (2004) show empirically that the marginal rate of substitution of consumption for money has a strictly positive lower bound. This result implies that the marginal utility of money is insatiable.

[16] I assume that the low-wage households pay taxes and save money.

Equations (36)-(38) form an autonomous dynamic system with respect to c_1, c_2 and m_2, provided that real values w_2 and z_2 are assumed to be constant, that is, $\frac{\dot{W}_2}{W_2} = \frac{\dot{Z}_2}{Z_2} = \pi^d$. Thereby, consumption c_1 and c_2, and real money balances m_2 are constant in the steady state equilibrium. The saddle path to the equilibrium can be realized under the following conditions (see in Appendix A.4).

$$2\rho + \pi^d - \frac{c_1^{d*}\pi_{c_1}^d}{\eta_{c_1}} - \frac{c_2^{d*}\pi_{c_2}^d}{\eta_{c_2}} > 0, \tag{39}$$

$$\rho + \pi^d - \frac{c_1^{d*}\pi_{c_1}^d}{\eta_{c_1}} > 0, \tag{40}$$

$$m_2^{d*}\pi^d + \frac{\eta_{m_2}c_2^{d*}}{\eta_{c_2}} > 0, \tag{41}$$

where $\eta_{m_2} \equiv -\frac{v''(m_2)m_2}{v'(m_2)} > 0$. (39) and (40) are satisfied under Assumption 1.

Assumption 1 *The adjustment speed of the disequilibrium of supply and demand in the goods market α is small enough to makes $\pi_{c_i}^d, (i = 1, 2)$ sufficiently small.*

Assumption 1 leads to very small $\pi_{c_i}^d$ and then (39) and (40) are realized. Further (41) is satisfied under Assumption 2.

Assumption 2 *The consumption elasticity of marginal utility is equal to or higher than the money elasticity of marginal utility, $\eta_{c_2} \geq \eta_{m_2}$, and the amount of after-tax income and saving money stock is positive in the neighborhood of the steady state, $w_2 - z_2 > 0$.*

$\eta_{c_2} \geq \eta_{m_2}$ includes the case that the functions $u(c_2)$ and $v(c_2)$ are log utility functions. Assumption 2 gives $m_2^{d*}\pi^d + \frac{\eta_{m_2}c_2^{d*}}{\eta_{c_2}} > m_2^{d*}\pi^d + c_2^{d*} = w_2 - z_2 > 0$ in the neighborhood of the steady state by using the budget constraint of the low-wage households. Thus, (41) is realized.

At the same time, in the steady state, m_1 continues to increase and the demand shortage cannot be resolved in spite of an ongoing deflation as shown in Ono (2001).[17]

$$\dot{m}_1 = -\frac{m^s\pi}{n_1^d} > 0. \tag{45}$$

[17]Since $\dot{m}_2 = 0$ in the steady state, the time differential of the money market equilibrium (14) becomes

$$\dot{m}^s = n_1^d\dot{m}_1. \tag{42}$$

Assuming that the gorvernment's money expansion is constant in (15), the money stock increases at the deflation (negative inflation) rate, that is, $\frac{\dot{m}^s}{m^s} = -\pi$. Hence, the change in the money holdings of high-wage households (42) can be written as (45). Further, (45) can be solved using the steady state valuables as

$$m_1(t) = -\frac{n_2^d}{n_1^d}\pi^d m_2^{d*}te^{-\pi t} + m_1(0), \tag{43}$$

where $m_1(0)$ indicates the initial ($t = 0$) money holding of high-wage job households. Accordingly, the

Therefore, the steady state equilibrium values are

$$c_i^{d*} = c_i^{d*}(w_2), \quad \frac{dc_i^{d*}}{dw_2} > 0, \quad (i = 1, 2) \tag{46}$$

$$m_2^{d*} = m_2^{d*}(w_2), \quad \frac{dm_2^{d*}}{dw_2} > 0, \tag{47}$$

$$\pi^{d*} = \pi^{d*}(w_2), \quad \frac{d\pi^{d*}}{dw_2} > 0, \tag{48}$$

The differential coefficients can be derived using Assumption 1 (see Appendix A.5). In the steady state equilibrium, a minimum wage hike raises an overall wage through the setting of firm's efficiency wage, and it alleviates the deflation caused by correcting supply and demand imbalances, because the minimum wage hike has a decreasing effect on potencial output level, $\frac{dy^s}{dw_2} < 0$ and an increasing effect on the aggregate demand, $\frac{d(c_1^{d*}n_1^d + c_2^{d*}n_2^d + g)}{dw_2} > 0$. At that time, the minimum wage hike induces the households to increase consumption and firms to increase employment.[18] Proposition 3 summarizes these results.

Proposition 3 *In a demand-shortage regime economy, an increase in the minimum wage raises the inflation rate and aggregate demand, and decreases the unemployment rate.*

Social welfare in the demand-shortage regime V^d can be expressed as

$$V^d = \frac{1}{\rho}\left[n_1^d u(c_1^{d*}) + n_2^d u(c_2^{d*}) + n_2^d v(m_2^{d*})\right] - \frac{n_2^{d*} m_2^{d*} \pi^{d*} v'(m_1(0))}{\rho^2}, \tag{49}$$

where $m_1(0)$ indicates the initial ($t = 0$) money holding of high-wage job households and the last term in (49) is derived from $\int_0^\infty n_1 v(m_1) e^{-\rho t} dt$. The derivation of (49) is presented in Appendix A.6. Differentiating V^d with w_2 gives the following Proposition 4.

Proposition 4 *In the demand-shortage regime economy, an increase in the minimum wage improves the social welfare.*

LHS of the transversality condition (13) is

$$\lim_{t\to\infty} u'(c_1)\left[-\frac{n_2^d}{n_1^d}\pi^d m_2^{d*} t e^{-\pi^d t} + m_1(0)e^{-\rho t}\right] = \lim_{t\to\infty} u'(c_1) m_1(0) e^{-\rho t} + \lim_{t\to\infty}\left[-u'(c_1)\frac{n_2^d}{n_1^d}\pi^d m_2^{d*} t e^{-(\rho+\pi^d)t}\right] \tag{44}$$

Since $-\frac{n_2^d}{n_1^d}\pi^d m_2^{d*} t e^{-(\rho+\pi^d)t}$ is monotonically decreasing $t \in (\frac{1}{\rho+\pi^d}, \infty)$, (44) converges to zero as $t \to \infty$. Thus, the transversality condition can be satisfied even under (45).

[18] In the initial steady state before the government raises the minimum wage, it sets the minimum wage in proportion to the inflation rate. To implement a minimum wage hike policy, the government raises the minimum wage level transiently. The economy goes to the new steady state equilibrium as the government sets the minimum wage in proportion to the inflation rate that is higher than initial steady state because in the new steady state equilibrium, the inflation rate becomes higher.

Since $c_1^{d*}, c_2^{d*}, n_2^{d*}, m_2^{d*}$, and π^{d*} are increasing functions of w_2, the increase in the minimum wage raises the value of the first bracket term. However, the last term has two opposite effects. The first is a positive effect, caused by the increase in n_2^{d*} and m_2^{d*}, because $\frac{-\pi^{d*}v'(m_1(0))}{\rho^2 m_1(0)} \frac{d(n_2^{d*}m_2^{d*})}{dw_2} > 0$ where $\pi^{d*} < 0$. The second effect is negative, because $-\frac{n_2^{d*}m_2^{d*}v'(m_1(0))}{\rho^2 m_1(0)} \frac{d\pi^{d*}}{dw_2} < 0$. Assumption 1 implies that the first positive effect dominates the second one, because Assumption 1 weakens the minimum wage effects on the inflation rate.

Proposition 3 and 4 are very important when policy director implements the minimum wage policy. They have to consider the economic situation very carefully, that is, whether the economy is experiencing demand shortage and deflation or not. If the economy faces the demand shortage, the minimum wage hike can raise the aggregate demand, employment and social welfare.

5 Conclusion

This paper analyzes the role of a minimum wage in macroeconomics with dynamic general equilibrium model giving rise to two different equilibria. The study demonstrates that a minimum wage hike has positive effects on an employment rate, aggregate consumption, and social welfare under a demand shortage economy whereas does not under a non-demand shortage economy. This implies that the policy director can improve the aggregate economic activity by the increase in the minimum wage without any government spending. However, the increase in the minimum wage may entails the unfavorable side effects that the natural rate of unemployment rate goes up. As for countries that faces the demand shortage or diminishing price pressure, the minimum wage policy can be considered as one of the effective option to stimulate economic activity.

To consider the more realistic minimum wage policy, the extension of incorporating the productivity growth rate may be desireble. Productivity growth affects the natural unemployment rate, the potential output level, the inflation rate and the wage growth rate. Under this situation, the growth rate of minimum wage should be examined as a policy tool.

A Appendix

A.1 Appendix 1 Saddle stability under the supply-side regime

I show the local saddle stability conditions of the dynamics described by (24)-(27). Linearizing these equations in the neighborhood of the steady state values $c_i^*, m_i^* \quad (i =$

1, 2) gives[19]

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \rho+\pi^*-\frac{c_1^*\pi_{c_1}}{\eta_{c_1}} & -\frac{c_1^*\pi_{c_2}}{\eta_{c_1}} & -\frac{v''(m_1)}{u''(c_1)} & 0 \\ -\frac{c_2^*\pi_{c_1}}{\eta_{c_2}} & \rho+\pi^*-\frac{c_2^*\pi_{c_2}}{\eta_{c_2}} & 0 & -\frac{v''(m_2)}{u''(c_2)} \\ -m_1^*\pi_{c_1}-1 & -m_1^*\pi_{c_2} & -\pi^* & 0 \\ -m_2^*\pi_{c_1} & -m_2^*\pi_{c_2}-1 & 0 & -\pi^* \end{bmatrix} \begin{bmatrix} c_1-c_1^* \\ c_2-c_2^* \\ m_1-m_1^* \\ m_2-m_2^* \end{bmatrix} \tag{50}$$

Noting that, for example, $\frac{\partial\left(\frac{v'(m_1^*)}{u'(c_1^*)\eta_{c_1}}\right)}{\partial c_1^*} = \frac{-u''(c_1^*)v'(m_1^*)}{(u'(c_1^*))^2\eta_{c_1}} = \frac{v'(m_1^*)}{u'(c_1^*)c_1^*} = \frac{\rho+\pi^*}{c_1^*}$ because I assume $\eta_{c_i} = \frac{-u''(c_i^*)c_i^*}{u'(c_i^*)}$ does not depend on c_i^*. While the real consumption of each household are jumpable valueables at any point in time, the real money balances of each household are not. Thus, for the system to have a stable saddle path, it must have two positive roots (or a pair of complex roots with a positive real part) and two negative real roots (or a pair of complex roots with a negative real part).

Denoting this Jacobian matrix as A^s, the eigenvalue equation can be expressed as

$$\lambda_s^4 - Trace\,(A^s)\,\lambda_s^3 + B^s\lambda_s^2 - C^s\lambda_s + det\,(A^s) = 0,$$

where

$$B^s = \rho^2 - \rho\left(\frac{c_1^*\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^*\pi_{c_2}}{\eta_{c_2}}\right) - \frac{v''(m_1^*)}{u''(c_1^*)}\left(1+m_1^*\pi_{c_1}\right) - \frac{v''(m_2^*)}{u''(c_2^*)}\left(1+m_2^*\pi_{c_2}\right),$$

$$det\,(A^s) = \left(\frac{v''(m_1^*)}{u''(c_1^*)}\right)^2\left(\frac{v''(m_2^*)}{u''(c_2^*)}\right)^2\left(1+m_1^*\pi_{c_1}+m_2^*\pi_{c_2}\right) > 0. \tag{51}$$

Letting λ_{sk}, $(k=1,2,3,4)$ be the roots of the eigenvalue equation. If the following conditions are satisfied at least, the system has a locally stable saddle point.[20]

$$\lambda_{s1}\lambda_{s2}\lambda_{s3}\lambda_{s4} > 0, \tag{52}$$

$$\lambda_{s1}\lambda_{s2} + \lambda_{s1}\lambda_{s3} + \lambda_{s1}\lambda_{s4} + \lambda_{s2}\lambda_{s3} + \lambda_{s2}\lambda_{s4} + \lambda_{s3}\lambda_{s4} < 0. \tag{53}$$

Considering the relation rule between roots and coefficients of the eigenvalue equation, the LHS of (52) and (53) equal $det\,(A^s)$ and B^s, respectively. (52) is satisfied already by $det\,(A^s) > 0$. $B^s < 0$ is satisfied if

$$\rho\left(\frac{c_1^*\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^*\pi_{c_2}}{\eta_{c_2}}\right) + \frac{v''(m_1^*)}{u''(c_1^*)}\left(1+m_1^*\pi_{c_1}\right) + \frac{v''(m_2^*)}{u''(c_2^*)}\left(1+m_2^*\pi_{c_2}\right) > \rho^2.$$

Under this condition, the transversality conditions (13) are satisfied.

[19] I here abbreviate the index s meaning supply side regime.

[20] Shimomura (2004) discusses saddle-path stability in dynamic general equilibrium model in greater detail.

A.2 Appendix 2 Derivation of (28) and (29)

Using the Cramer's rule in the steady state equilibrium, (28) is

$$\frac{dc_1^*}{dw_2} = \frac{1}{det\,(A^s)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^*\pi_{c_2} - (1+m_2^*\pi_{c_2})\frac{n_2^s}{n_1^s} - m_1^*\pi_{w_2}\right] < 0,$$
$$\frac{dc_2^*}{dw_2} = \frac{1}{det\,(A^s)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[1+m_1^*\pi_{c_1} + m_2^*\pi_{c_1}\frac{n_2^s}{n_1^s} - m_2^*\pi_{w_2}\right]$$

where $det\,(A^s) > 0$ as shown in (51). Furthermore, the effects of a minimum wage hike on aggregate consumption is

$$\begin{aligned}\frac{d(n_1^s c_1^* + n_2^s c_2^*)}{dw_2} &= \frac{dc_1^*}{dw_2}n_1^s + \frac{dn_1^s}{dw_2}c_1^* + \frac{dc_2^*}{dw_2}n_2^s + \frac{dn_2^s}{dw_2}c_2^* \\ &= \frac{1}{det\,(A^s)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^*n_1^s\pi_{w_2} - m_2^*n_2^s\pi_{w_2} + \frac{dn_2^s}{dw_2}c_2^*\right] < 0,\end{aligned}$$

where $\frac{dn_1^s}{dw_2} = 0$ and $\frac{dn_2^s}{dw_2} < 0$ as shown in (20) and (21).[21]

Differentiating the steady state equilibrium condition $\frac{v'(m_i^*)}{u'(c_i^*)} = \rho$ with w_2 yields $\frac{u''(c_i^*)c_i^*}{u'(c_i^*)}\frac{dc_i^*}{dw_2}\frac{w_2}{c_i^*} = \frac{v''(m_i^*)m_i^*}{v'(m_i^*)}\frac{dm_i^*}{dw_2}\frac{w_2}{m_i^*}$, and then

$$\frac{dm_i^*}{dw_2} = \frac{\rho u''(c_i^*)}{v''(m_i^*)}\frac{dc_i^*}{dw_2}.$$

A.3 Appendix 3 Proof of Proposition 1

In the steady state, differentiating (30) with w_2 yields

$$\begin{aligned}\frac{dV^s}{dw_2} &= \frac{1}{\rho}\left[u'(c_1^*)\frac{dc_1^*}{dw_2}n_1^s + v'(m_1^*)\frac{dm_1^*}{dw_2}n_1^s + u'(c_2^s)\frac{dc_2^*}{dw_2}n_2^s + v'(m_2^*)\frac{dm_2^*}{dw_2}n_2^s + \frac{dn_2^s}{dw_2}\left(u(c_2^*) + v(m_2^*)\right)\right] \\ &= \frac{1}{\rho}\left[\frac{dc_1^*}{dw_2}n_1^s\left(u'(c_1^*) + \rho\frac{u''(c_1^*)}{v''(m_1^*)}\right) + \frac{dc_2^*}{dw_2}n_2^s\left(u'(c_2^*) + \rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \frac{dn_2^s}{dw_2}\left(u(c_2^*) + v(m_2^*)\right)\right].\end{aligned}$$

Defining $u'(c_1^*) + \rho\frac{u''(c_1^*)}{v''(m_1^*)} = u'(c_2^*) + \rho\frac{u''(c_2^*)}{v''(m_2^*)} + \varepsilon$ yields

$$\frac{dV^s}{dw_2} = \frac{1}{\rho}\left[\left(\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s\right)\left(u'(c_2^*) + \rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \varepsilon\frac{dc_1^*}{dw_2}n_1^s + \frac{dn_2^s}{dw_2}\left(u(c_2^*) + v(m_2^*)\right)\right].$$

Noting that $\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s = \frac{1}{det(A^s)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^*n_1^s\pi_{w_2} - m_2^*n_2^s\pi_{w_2}\right] < 0$, the condition of $\frac{dV^s}{dw_2} < 0$ is given by

$$\varepsilon > \frac{-1}{\frac{dc_1^*}{dw_2}n_1^s}\left[\left(\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s\right)\left(u'(c_2^*) + \rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \frac{dn_2^s}{dw_2}\left(u(c_2^*) + v(m_2^*)\right)\right].$$

[21] In the derivation of aggregate consumption, I have used $\pi_i = \frac{-\alpha n_i}{y^s}$.

A.4 Appendix 4 Saddle stability under the demand-shortage regime

I show the local saddle stability conditions of the dynamics described by (36)-(38). Linearizing these equations in the neighborhood of the steady state values c_1^*, c_2^*, m_2^* gives[22]

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \rho + \pi^d - \frac{c_1^* \pi_{c_1}}{\eta_{c_1}} & -\frac{c_1^* \pi_{c_2}}{\eta_{c_1}} & 0 \\ -\frac{c_2^* \pi_{c_1}}{\eta_{c_2}} & \rho + \pi^d - \frac{c_2^* \pi_{c_2}}{\eta_{c_2}} & -\frac{v''(m_2)}{u''(c_2)} \\ -m_2^* \pi_{c_1} & -m_2^* \pi_{c_2} - 1 & -\pi^d \end{bmatrix} \begin{bmatrix} c_1 - c_1^* \\ c_2 - c_2^* \\ m_2 - m_2^* \end{bmatrix}. \tag{54}$$

The real consumption of households are jumpable valuables at any point in time, but the real money balances of low-wage job households cannot jump. For the system to have a stable saddle path, it must have two positive roots (or a pair of complex roots with a positive real part) and one negative real root (or a pair of complex roots with a negative real part). Defining this Jacobian matrix as A^d, the trace and determinant of A^d are given by

$$Trace\left(A^d\right) = 2\rho + \pi^d - \left(\frac{c_1^* \pi_{c_1}}{\eta_{c_1}} + \frac{c_2^* \pi_{c_2}}{\eta_{c_2}}\right),$$
$$det\left(A^d\right) = (\rho + \pi^d)\left[\left(\rho + \pi^d - \frac{c_1^* \pi_{c_1}}{\eta_{c_1}}\right)\left(\frac{-1}{m_2^*}\right)\left(m_2^* \pi^d + \frac{\eta_{m_2} c_2^*}{\eta_{c_2}}\right) + \frac{c_2^* \pi_{c_2}}{\eta_{c_2}}\left(\pi^d - \eta_{m_2}(\rho + \pi^d)\right)\right].$$

If both $det\left(A^d\right) < 0$ and $Trace\left(A^d\right) > 0$ are satisfied at least, the system has a locally stable saddle point. The condition (39) is obtained easily by $Trace\left(A^d\right) > 0$. π^d is negative in the neighborhood of the steady state because demand shortage arises. $\rho + \pi^d$ is positive because marginal rate of substitution of real money balances and consumption is positive, and then the last term of $det\left(A^d\right)$ is negative. Therefore, if conditions (40) and (41) are satisfied, $det\left(A^d\right) < 0$.

A.5 Appendix 5 derivation of equilibrium values in the demand shortage regime

In the steady state equilibrium (36)-(38) are

$$\beta = (\rho + \pi^d) u'(c_1^{d*}),$$
$$v'(m_2^{d*}) = (\rho + \pi^d) u'(c_2^{d*}),$$
$$m_2^{d*} \pi^d = w_2 - z_2 - c_2^{d*}.$$

Total differentials of these equations are

$$\eta_{c_1} \frac{dc_1}{c_1} = \frac{d\pi^d}{\rho + \pi^d}, \tag{55}$$

[22] I abbreviate the index d, meaning demand-shortage regime, here.

$$u''(c_2)dc_2 + u'(c_2)d\pi^d = v''(m_2)dm_2, \tag{56}$$

$$\pi^d dm_2 + m_2 d\pi^d = dw_2 - dc_2, \tag{57}$$

where $dz = 0$ because the effect of z is not analyzed.

On the other hand, the total differential of (35) is

$$d\pi^d = \pi^d_{c_1} dc_1 + \pi^d_{c_2} dc_2 + \pi^d_{w_2} dw_2. \tag{58}$$

Combining (55) and (58) yields

$$\left(\eta_{c_1} - \frac{\pi^d_{c_1} c_1}{\rho + \pi^d}\right) \frac{dc_1}{c_1} = \frac{\pi^d_{c_2} dc_2}{\rho + \pi^d} + \frac{\pi^d_{w_2} dw_2}{\rho + \pi^d}, \tag{59}$$

Assumption 1 gives the positive value of the first parenthesis of the LHS in (59), and then

$$c_1 = c_1(c_2, w_2), \quad \frac{\partial c_1}{\partial c_2} > 0, \quad \frac{\partial c_1}{\partial w_2} > 0, \tag{60}$$

$$\pi^d = \pi^d(c_1(c_2, w_2), c_2, w_2) = \pi^{dd}(c_2, w_2), \quad \pi^{dd}_{c_2} = \frac{\partial \pi^{dd}}{\partial c_2} > 0, \quad \pi^{dd}_{c_2} = \frac{\partial \pi^{dd}}{\partial w_2} > 0. \tag{61}$$

Furthermore, combining (56), (57), and (61) yields

$$\left[m_2 \left(1 - \frac{\pi^d}{\eta_{m_2}(\rho + \pi^d)}\right) \pi^{dd}_{c_2} + \frac{\eta_{m_2}}{\eta_{c_2} c_2} \left(m_2 \pi^d + \frac{\eta_{m_2}}{\eta_{c_2}} c_2\right)\right] dc_2$$
$$= \left[1 - m_2 \left(1 - \frac{\pi^d}{\eta_{m_2}(\rho + \pi^d)}\right) \pi^{dd}_{w_2}\right] dw_2. \tag{62}$$

Assumption 1 gives the sufficiently small values of $\pi^{dd}_{c_2}$ and $\pi^{dd}_{w_2}$, and Assumption 2 gives the $m_2 \pi^d + \frac{\eta_{m_2}}{\eta_{c_2}} c_2 > 0$. These conditions lead to

$$c_2^{d*} = c_2^{d*}(w_2), \quad \frac{dc_2^{d*}}{dw_2} > 0. \tag{63}$$

Substituting (63) into (60) and (61) yields

$$c_1^{d*} = c_1^{d*}(w_2), \quad \frac{dc_1^{d*}}{dw_2} > 0, \tag{64}$$

$$\pi^{d*} = \pi^{d*}(w_2), \quad \frac{d\pi^{d*}}{dw_2} > 0. \tag{65}$$

Combining (56), (64), and (65) with Assumption 1 yields

$$m_2^{d*} = m_2^{d*}(w_2), \quad \frac{dm_2^{d*}}{dw_2} > 0. \tag{66}$$

A.6 Appendix 6 Derivation of (49)

The last term in (49) can be derived as follows.

$$\begin{aligned}
\int_0^\infty n_1^d v(m_1(t))e^{-\rho t}dt &= n_1^d\left[\frac{e^{-\rho t}v(m_1(t))}{-\rho}\right]_0^\infty - n_1^d\int_0^\infty\left[\frac{e^{-\rho t}v'(m_1(t))m_1'(t)}{-\rho}\right]dt \\
&= \frac{n_1^d v(m_1(0))}{\rho} - n_1^d\left[\frac{e^{-\rho t}v(m_1(t))}{-\rho} + \frac{e^{-\rho t}v'(m_1(t))m_1'(t)}{\rho^2}\right]_0^\infty \\
&= \frac{n_1^d v(m_1(0))}{\rho} + \frac{n_1^d}{\rho}\left[\frac{v'(m_1(0))m_1'(0)}{\rho} - v(m_1(0))\right] \\
&= \frac{n_1^d v'(m_1(0))m_1'(0)}{\rho^2} = \frac{n_1^d v'(m_1(0))\left(\frac{-n_2^d m_2^{d*}\pi^{d*}}{n_1^d}\right)}{\rho^2},
\end{aligned}$$

where $m_1'(0) = \frac{-n_2^d m_2^{d*}\pi^{d*}}{n_1^d}$ because $m_1(t) = m_1(0) + \frac{n_2^d}{n_1^d}\pi^{d*}m_2^{d*}te^{-\pi t})e^{-\rho t}$ as in (43), which can be differentiated with t, and hence, $m_1'(t) = (t\pi^{d*} - 1)\frac{\pi^{d*}n_2^d m_2^{d*}e^{-\pi^{d*}t}}{n_1^d}$. Substituting $t = 0$ with this equation yields $m_1'(0) = \frac{-n_2^d m_2^{d*}\pi^{d*}}{n_1^d}$.

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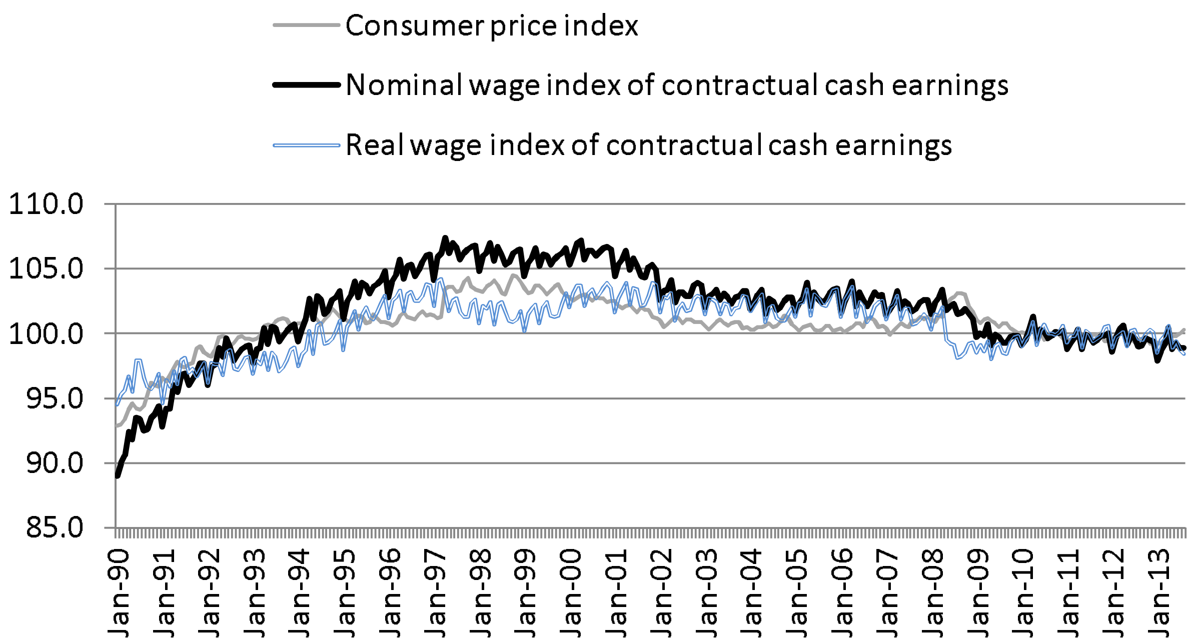

Fig 1: CPI and wage in Japan 1990-2013.

Notes and sources: The source for CPI is the consumer price index conducted every month by the Statistics Bureau, Ministry of Internal Affairs and Communications. The source for nominal and real wage index is the 2010-base index of "Monthly Labour Survey" conducted every month by the Ministry of Health, Labour and Welfare. Contractual cash earnings is defined as the amount before deducting income tax and social insurance premium.