From Oldham's Coupling to Air Conditioners

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Abstract

Oldham's coupling is a really interesting device. It is possible to understand the method with a knowledge of only junior high-school geometry. Compressors in air conditioners also use this idea, where involute curves are used for the teeth of the cogs in the compressor. Mathematics doesn't just reside in textbooks; it's alive in our daily lives.

Keywords: Oldham's coupling, Turning block double-slider mechanism,
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1. In a certain museum ...

Mathematics is not just to be found in textbooks, it exists in our daily lives. It does not just have abstract forms, but is also tangible and constant. Once when I was visiting Kyoto University Museum and wondering whether there were any interesting educational materials, my eyes fell upon a model of a device known as Oldham's coupling and it caught my attention. Oldham's coupling is a design that was imported into Japan through Germany at the time of modernization in the 19^{th} and 20^{th} centuries. Oldham is apparently the name of the man who devised it. I was able to touch the object, and I was captivated by its peculiar and wondrous motion.

It has two parallel axes, which are slightly offset. How can the rotation of the left axis be correctly transmitted to the rotation of right axis? The idea of using three cogs may occur to an amateur. There would be two cogs with the same number of teeth, and one for changing the direction of motion. A solution is thus possible with a total of three cogs. However, when the axial distance is too small, correspondingly tiny cogs are required, which is not realistic. There is the so-called 'universal joint' used in cars, and while it is possible to solve the problem in this way, the device ends up being rather complicated. It is also possible to apply a belt, but belts stretch, shrink, and wear down, so the rotation is not transmitted correctly. 'Oldham's coupling' which I will introduce here, is extremely mathematical, but it does not require high level mathematics. Rather, it is possible to understand the method with a knowledge of only junior high-school geometry.

2. Transmitting rotation between parallel axes

The structure of Oldham's coupling is shown in Figure 1, which is a reproduction based on the work of Hitoshi Morita (see Morita, 1974).

It is composed of 3 discs, a, b and c. Applying a rotation to disc a or c causes disc b to rotate while sliding with respect to a and c. This mechanism is known as a turning block double-slider