Graduate School of Economics, Osaka University of Economics

Working Paper Series

No.2024-01

TRENDS AND RANDOM WALK IN MACROECONOMIC TIME SERIES: THE CASE OF HONG KONG SPECIAL ADMINISTRATIVE REGION, CHINA

大阪経済大学大学院　経済学研究科　博士後期課程

NGUYEN NGOC MAI

2024 年 11 月

Graduate School of Economics, Osaka University of Economics
Working Paper Series
No.2024-01

TRENDS AND RANDOM WALK IN MACROECONOMIC TIME SERIES: THE CASE OF HONG KONG SPECIAL ADMINISTRATIVE REGION, CHINA

Nguyen Ngoc Mai[1]

ABSTRACT

This study aims to examine the stationarity of key macroeconomic variables in Hong Kong Special Administrative Region, China (Hong Kong SAR). Nelson and Plosser (1982) argued that most macroeconomic variables in the United States follow a difference stationary process, except for the unemployment rate. Building on their work, it is suggested that real GDP, nominal GDP, and employment in Hong Kong follow a random walk with a drift. The results of the F-test indicate that real GDP, nominal GDP, and employment indeed appear to follow a random walk with a drift, while the unemployment rate follows a simple random walk. In contrast, the CPI shows evidence of being trend stationary. Additional analysis, including the sample autocorrelation and the ADF test, further supports that the CPI follows a trend stationary process.

Keywords: difference stationary, trend stationary, random walk, Hong Kong SAR, China

JEL Classification: E24, E31, E32

[1]Corresponding author: Graduate School of Osaka University of Economics, Email: ngocmainguyen1922@gmail.com

1. INTRODUCTION

This study aims to examine the stationarity of key macroeconomic variables in Hong Kong Special Administrative Region, China (Hong Kong SAR). Nelson and Plosser (1982) argued that macroeconomic variables in the United States (U.S.) generally follow a difference stationary process, with the unemployment rate being an exception. Building on their approach, we investigate the stationarity of key macroeconomic variables in Hong Kong SAR.

Nelson and Plosser (1982) summarize their findings as follows:

1. Real shocks associated with the secular component significantly contribute to variations in observed output.

2. These shocks are either correlated with innovations in the cyclical component, or the secular component contains transitory fluctuations—or both.

Campbell and Mankiw (1987) estimated standard ARIMA models for the log of U.S. real GNP using post-war quarterly time series. They suggested that a 1 percent innovation in real GNP could change long-term forecasts of real GNP by more than 1 percent. Their analysis supports the persistence of shocks, aligning with the evidence presented by Nelson and Plosser and their own findings, which is consistent with real business cycle models. Blanchard and Fischer (1989) observed that, despite short-run fluctuations, most economists believe the economy evolves along an underlying growth path—often referred to as the trend. Similarly, Campbell and Mankiw (1987) argued that it cannot be ruled out that some fraction of an innovation in economic series is permanent. This implies that shocks to these variables have a permanent impact, without reverting to a long-term mean or trend. Drawing further inspiration from Nelson and Plosser (1982), Kurosaka (2024) extended this line of inquiry by analyzing the stationarity of real wages during Japan's ruling period in the Korean Peninsula. Motivated by these arguments, this study investigates whether key macroeconomic variables in Hong Kong SAR follow a difference stationary or trend stationary process.

This paper is organized as follows: Section 2 describes the data sources and processing methods, along with time series plots of the main macroeconomic variables in their natural logarithms. Section 3 presents the analysis of the autocorrelation function, as well as the results of the ADF and F-tests for the key macroeconomic variables. Finally, Section 4 concludes with a summary of the key findings and their implications.

2. SOURCES OF DATA

The data for this study was sourced from the Census and Statistics Department of the

Hong Kong SAR Government. It includes quarterly data on Hong Kong's Real Gross Domestic Product (RGDP), Nominal Gross Domestic Product (NGDP), Unemployment Rate (UR), Employment (EMP), and Composite Consumer Price Index (CPI) spanning from the first quarter of 1985 to the fourth quarter of 2023[2]. Using the method described in Nguyen (2023), the quarterly data were seasonally adjusted. The analysis also uses data from the first quarter of 1986 to the fourth quarter of 2022. All data have been transformed into natural logarithms. Figures 1 through 5 illustrate the natural logarithms of these variables.

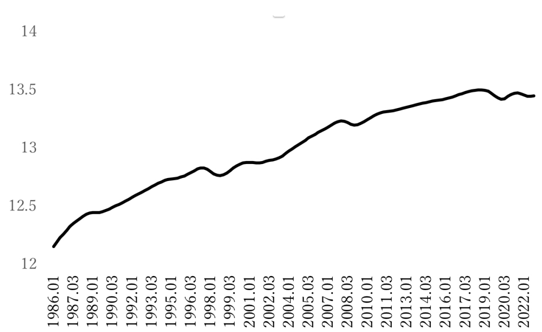

Figure 1: Natural logarithm of real GDP

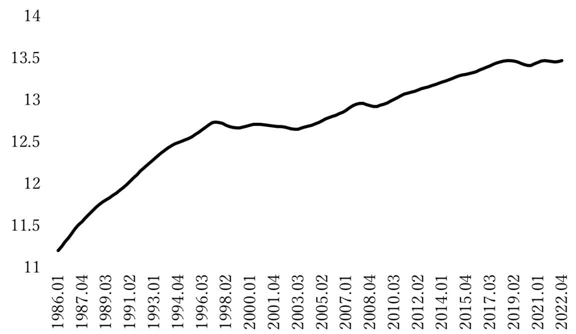

Figure 2: Natural logarithm of nominal GDP

Figure 3: Natural logarithm of unemployment rate

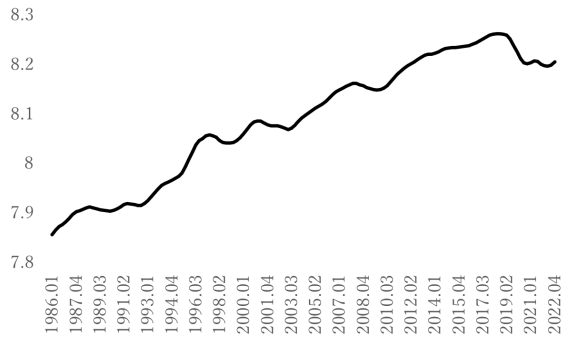

Figure 4: Natural logarithm of employment

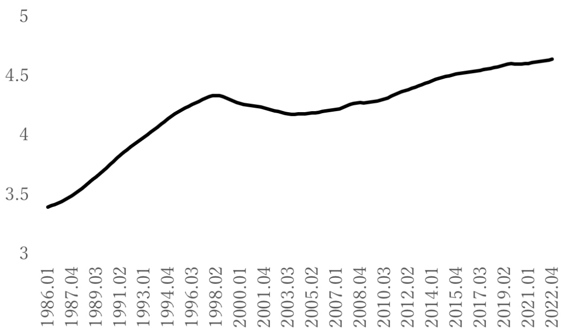

Figure 5: Natural logarithm of consumer price index

[2]Available from: http://www.censtatd.gov.hk/en/

Figures 1, 2, 4, and 5 reveal an upward trend over time in these series. In contrast, Figure 3 shows that the unemployment series does not exhibit a clear trend.

3. ANALYSIS OF THE MAIN MACROECONOMICS IN HONG KONG SAR

3.1. Analysis of the sample autocorrelation

Following the methodology outlined in Nelson and Plosser (1982), we calculated the autocorrelation function of the variables for lags 1 through 6, as presented in the tables below. Table 1 reports the autocorrelations of the natural logarithms of the main macroeconomic variables. Table 2 displays the autocorrelations of the first-order differences of the natural logarithms of all variables. Table 3 presents the autocorrelations of the deviations from the fitted time trend using linear detrending methods.

Table 1: Autocorrelations of Various Economic Variables

Variable	r_1	r_2	r_3	r_4	r_5	r_6
RGDP	0.977	0.955	0.933	0.912	0.891	0.870
NGDP	0.969	0.939	0.909	0.879	0.850	0.821
UR	0.970	0.929	0.879	0.823	0.766	0.717
EMP	0.982	0.964	0.948	0.931	0.912	0.893
CPI	0.971	0.941	0.910	0.878	0.846	0.813

Note: r_i refers to the i-th order autocorrelation of the series.

Table 2: Autocorrelations of the First Difference of the Natural Logs

Variable	$r(1)$	$r(2)$	$r(3)$	$r(4)$	$r(5)$	$r(6)$
RGDP	0.875	0.646	0.393	0.155	0.001	-0.062
NGDP	0.936	0.817	0.684	0.554	0.460	0.407
UR	0.182	0.142	0.095	0.006	-0.143	-0.197
EMP	0.885	0.661	0.422	0.215	0.087	-0.006
CPI	0.983	0.943	0.897	0.850	0.810	0.774

Note: $r(i)$ refers to the i-th order autocorrelation of the first difference of the series.

Table 3: Autocorrelations of the Deviations from the Linear Time Trend

Variable	$d(1)$	$d(2)$	$d(3)$	$d(4)$	$d(5)$	$d(6)$
RGDP	0.928	0.845	0.758	0.673	0.597	0.534
NGDP	0.953	0.904	0.852	0.800	0.749	0.701
UR	0.954	0.895	0.828	0.761	0.690	0.634
EMP	0.934	0.861	0.775	0.683	0.599	0.520
CPI	0.978	0.952	0.922	0.887	0.849	0.808

Note: $d(i)$ refers to the i-th order autocorrelation coefficient for the detrended values of the series.

The following discussion is based on Enders (2015). Table 1 shows that the autocorrelations of the natural logarithms of the variables remain close to 1 from lags 1 to 4 and decay slowly with increasing lags, suggesting a unit root process. Table 2 demonstrates that the first differences of the natural logarithms of real GDP, nominal GDP, employment, and CPI show high autocorrelation values up to the second lag. For the unemployment rate positive autocorrelation at lag 1 suggests an MA (1) process, as noted by Enders (2015). Based on these findings, it is challenging to conclusively determine whether the differenced series of real GDP and nominal GDP are difference stationary. Additionally, the first difference of the CPI shows autocorrelations close to 1, raising the possibility that the consumer price index follows a trend stationary process.

Table 3 shows that the first, second, and third autocorrelation coefficients for the detrended values of the series remain close to 1. As Enders (2015) explains, this is consistent with the idea that detrending a difference stationary process does not eliminate non-stationarity. Enders also notes that differencing a trend-stationary process results in a non-invertible MA process. When the MA component includes a unit root, the autocorrelation coefficients of the differenced series do not decay. The lack of decay in the autocorrelation coefficients for the differenced series of the Consumer Price Index suggests the possibility of a trend-stationary process with a unit root in the MA component.

3.2. Augmented Dickey - Fuller test (ADF test)

Next, we conduct an ADF test to examine the presence of a unit root in the main macroeconomic variables of Hong Kong SAR. Following the approaches of Nelson and Plosser (1982) and Enders (2015), we assume that the time series of macroeconomic variables can be appropriately modeled using the following equation.

$$\Delta y_t = \beta y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-1-i} + \epsilon_t \quad (1) \text{ (Without constant term and a time trend)}$$

$$\Delta y_t = \mu + \beta y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-1-i} + \epsilon_t \quad (2) \text{ (With constant term)}$$

$$\Delta y_t = \mu + \delta t + \beta y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-1-i} + \epsilon_t \quad (3) \text{ (With a constant and a time trend)}$$

In these formulas, Δy_t represents the differenced series of main macroeconomic variables, and y_{t-1} is the lagged value of the series. The constant term is denoted by μ, while δ represents the coefficient of the trend. The coefficient β is associated with each approach, and ϵ_t denotes the error (white noise).

As noted earlier, the time series data for real GDP, nominal GDP, employment, and the consumer price index exhibit an upward trend. According to Enders (2015), such data can either be trend-stationary or contain a unit root with a drift term. Therefore, for these variables, it is necessary to estimate equation (3) and test the constraints $\beta = 0$ and $\mu = \delta = 0$. For the unemployment rate, which does not display an upward trend, equation (2) is estimated instead. Enders (2015) found that variables like real GNP and nominal GNP tend to exhibit difference stationarity, while the unemployment rate demonstrates trend stationarity. The results of this analysis are presented in Table 4.

Table 4: Test for Autoregressive Unit Roots

Variable	T	p	μ	δ	β	$\beta + 1$
RGDP	147	10	0.046	0.00002	-0.004	0.996
			(0.818)	*(0.463)*	*(-0.775)*	
NGDP	147	10	0.076	0.00006	-0.006	0.994
			(2.875)	*(2.467)*	*(-2.834)*	
UR	147	1	0.0438	-	*(-0.036)*	0.964
			(1.67)	-	*(1.798)*	
EMP	147	11	0.048	0.00001	-0.006	0.994
			(1.062)	*(0.790)*	*(-1.048)*	
CPI	147	9	0.022	0.00003	(-0.006***)	0.994
			(5.145)	*(4.528)*	*(-5.112)*	

Notes: T represents the number of observations. p is the chosen lag length. The value in parentheses is the t-statistic of the ordinary least squares estimates of μ, δ, and β. An (***) denotes significance at the 0.01 level.

As noted by Enders (2015), Dickey and Fuller (1979) found that the critical values for testing $\beta = 0$ depend on the regression form and sample size. Enders (2015) highlights the traditional view of business cycles, which holds that GNP and production levels are trend-stationary rather than difference stationary. According to this perspective, β must be significantly different from zero. If $\beta = 0$, the series has a unit root and is considered difference stationary. Based on the sample sizes used by Nelson and Plosser (1982), the critical value of the t-statistic for the null hypothesis $\beta = 0$ at the 0.05 significance level is -3.45.

The critical values for the t-statistic are -4.04, -3.45, and -3.15 at the 0.01, 0.05, and 0.10 significance levels, respectively. To examine stationarity, we compare the t-statistic of the coefficient β to these critical values. As shown in Table 4, the t-statistic for CPI is -5.11, which is below the critical value at the 0.01 level. This allows us to reject the null hypothesis that $\beta = 0$, indicating stationarity in the CPI series. In contrast, the estimated values of β for real GDP, nominal GDP, and employment are not statistically different from zero. Therefore, these variables do not exhibit stationarity. Table 4 also presents the estimation results for the unemployment rate based on equation (2). In this case, the null hypothesis $\beta = 0$ cannot be rejected, suggesting that the unemployment rate follows a difference stationary process.

3.3. F-test

As discussed by Enders (2015), Dickey and Fuller (1981) introduced three additional F-statistics (Φ_1, Φ_2, and Φ_3) to test joint hypotheses on the coefficients. As mentioned in Section 2, real GDP, nominal GDP, EMP, and CPI exhibit an upward trend, so these variables require the use of Φ_2 and Φ_3 statistics. Since the unemployment rate does not show a clear trend, only the Φ_1 statistic needs to be calculated. To test the null hypothesis $\beta = \mu = 0$, which assumes the data are generated by equation (1), against the alternative that equation (2) is the "true" model, we use the Φ_1 statistic. To test the null hypothesis $\beta = \delta = \mu = 0$, which assumes the data are generated by equation (1), against the alternative that equation (3) is the "true" model, we use the Φ_2 statistic. Lastly, to test the null hypothesis $\beta = \delta = 0$, which assumes the data are generated by equation (2), against the alternative that equation (3) is the "true" model, we use the Φ_3 statistic. Subsequently, we obtain the results summarized in Table 5.

Table 5: The result of F – test of each variable.

Variable	Φ_1	Φ_2	Φ_3
RGDP	-	1.66	0.12
NGDP	-	3.11	3.31
EMP	-	1.44	0.34
CPI	-	12.96***	15.96***
UR	1.4	-	-

Since the Φ_2, and $\Phi_3$statistics for the three variables are all smaller than the critical values calculated by Dickey and Fuller (1981), we cannot reject the null hypotheses. Therefore, we conclude that the real GDP, nominal GDP, and employment series behave as a random walk with drift. Similarly, the Φ_1 statistic for the unemployment rate is smaller than the critical value, meaning we cannot reject the null hypotheses. This suggests that the unemployment rate follows a random walk model. However, for CPI, the values of Φ_2 (12.96) and Φ_3 (15.96) exceed the critical value at the 1% significance level. Based on Enders' (2015) argument, we can reject the null hypothesis of a random walk in favor of the alternative hypothesis, which suggests that the data include an intercept, a unit root, and/or a deterministic time trend.

4. CONCLUSION

We conducted three types of analysis—sample autocorrelation, the Augmented Dickey-Fuller (ADF) test, and the F-test—to examine the stationarity of key macroeconomic variables in the Hong Kong SAR, China. Specifically, real GDP, nominal GDP, employment, and the unemployment rate appear to follow a random walk with a drift term, while the composite CPI seems to follow a trend stationary process. Although temporary shocks may cause deviations from the trend, the CPI ultimately returns to its deterministic path over time.

As noted by Enders (2015), special care must be taken when performing unit root tests if there is a suspicion of structural change. Structural breaks can bias the various Dickey-Fuller test statistics toward failing to reject the presence of a unit root. For future research, we plan to apply the Perron test, which accounts for structural changes, to the variables exhibiting unit roots in this study.

Theoretically, it is important to consider the following point: Mankiw and Campbell (1987) suggested that there are various mechanisms through which aggregate demand shocks could have permanent or near-permanent effects on the level of output. These shocks may have lasting impacts if technological innovation is influenced by the business cycle. This is also a theme for future research.

References

Campbell, J.Y., and Mankiw, N.G., (1987) “Are Output Fluctuations Transitory?” *The Quarterly Journal of Economic*, Nov., Vol. 102, No. 4, pp.857-880.

Blanchard, O. J. , and Fischer, S., (1989) “Lectures on macroeconomics”, *MIT press, Cambridge*

Dickey, D. A., Fuller, W. A. (1979). “Distribution of estimators for autoregressive time series with a unit root” *Journal of the American Statistical Association*, Volume 74, Issue 366, 427–431.

Dickey, D. A., Fuller, W. A. (1981). “Likelihood ratio statistics for autoregressive time series with a unit root”. *Econometrica*, Volume 49, No.4, 1057-1072

Enders.W., (2015). “Applied Econometric Time Series”. *John Wiley & Sons, Inc*, Fourth edition.

Kurosaka, M., (2024). “Stationarity of Real Wage During Japan’s Rulling Period over the Korean Peninsula” *Osaka University of Economics Working Paper*. Series No. 2024-1.

Nelson, C. R., & Plosser, C. I. (1982). "Trends and Random Walks in Macroeconomic Time Series." *Journal of Monetary Economics*, 10(2), 139-162.

Nguyen, N. M., (2023). “Real Wage Cyclicality in Hong Kong SAR” *Osaka University of Economics Working Paper*. Series No. 2023-1.