Osaka University of Economics Working Paper Series
No.2007-1

The Organization as a Rule of Intrafirm Bargaining

Fumi Kiyotaki
and
Toshiji Miyakawa

May, 2007

The Organization as a Rule of Intrafirm Bargaining*

Fumi Kiyotaki†
and
Toshiji Miyakawa‡

May 10, 2007

Abstract

In this paper, we study the choice of organizational form under incomplete contracts. We identify an organizational form with a rule of expost bargaining and compare four types of organization: horizontal organizations (partnerships); common agencies; pyramidal hierarchies; and vertical hierarchies. We show that if the human capital investments of all members are complementary and essential to production, the horizontal organization is chosen. If the investments of two players including the owner are essential, then the common agency may be optimal. If the pyramidal hierarchy can motivate subordinates to invest, the pyramidal hierarchy is chosen. The vertical hierarchy may be chosen if the owner can motivate a player who engages in firm-specific investment by assigning him or her to the middle rank. We also examine who should be assigned to the middle tier in the vertical hierarchy.

*We would like to thank Hideshi Itoh, Haruo Imai, Akira Okada and seminar participants at Hitotsubashi University and Konan University for useful comments.

†School of Economics, Kinki University, 3-4-1 Kowakae, Higashiosaka, Osaka 577-8502, Japan. Tel:+81 6 6721 2332 E-mail: kiyotaki@eco.kindai.ac.jp

‡Department of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa, Osaka 533-8533, Japan. Tel:+81 6 6328 2431. E-mail: miyakawa@osaka-ue.ac.jp

1 Introduction

The multilevel pyramidal hierarchy is widespread in large firms and many researchers have explained the rationale for the hierarchical form. However, there are other organizational forms in the real world. For example, it is well known that law firms adopt the partnership (horizontal organization). There are often two bosses (a common agency) in newly established firms. For example, Yahoo! and Google were founded by two people. Soichiro Honda, who was a founder of one of the biggest automobile companies, Honda, concentrated on the technology sector and his business partner, Takeo Fujisawa, engaged in management. Furthermore, there are hierarchical organizations that have a steep structure and those that have a flat structure.

Why are there various forms of organization in the real world? We focus on the incentives to invest in human capital under incomplete contracts and show that the characteristics of investments made by members determine the optimal organizational form for the owner of the firm.

The return on human capital investment is divided among the members of the organization following the investment because of the incompleteness of the contract. The bargaining position of each member over the returns from human capital investments differs between organizational forms. The bargaining power of the owner depends on whether only one person owns the firm, whether two people own the firm or whether all members collectively own the firm. Workers in the higher hierarchical rank may have stronger bargaining positions than workers in the lower ranks. The bargaining power of a worker depends on whether he or she has subordinates. Therefore, organizational structure can be regarded as an allocation of bargaining power. We identify an organizational structure with a rule of intrafirm bargaining.

According to studies of social (organization) power, French and Raven (1959) proposed five sources of power in an organization: (1) *Legitimate power*; (2) *Reward power*; (3) *Coercive power*; (4) *Expert power*; and (5) *Reference power*. We focus on (1) and (4). Legitimate power is based on the formal positions in organizations. Consider the army, or a bureaucratic hierarchy, for example. A superior can command his or her subordinates about what to do in the hierarchy. This means that the superior has greater formal bargaining power over his or her subordinates. Expert power is based on special knowledge or expertise in a given area. We suppose that each player can gain expert power by making human capital investments. From this viewpoint, we comment on how the relationship between legitimate power and expert power affects the choice of organizational form.

We model the situation described above by considering an organization of three players and by comparing four types of organizational form: the *hori-*

zontal organization, in which all members have equal authority; the *common agency*, in which there are two bosses; the *pyramidal hierarchy*, in which there is one boss and two subordinates of the same rank; and the *vertical hierarchy*, in which there is one boss, one supervisor and one subordinate. We analyze how the choice of organizational form determines the expost bargaining rule and affects the incentive to undertake human capital investments. The bargaining procedure is as follows: a player in the higher tier is able to make a proposal before a player in the lower tier can. If there is more than one player in the top rank, each is selected as the proposer with equal probability. A player makes a take-it-or-leave-it offer to his or her subordinates. We call a player who chooses the organizational form "player 1", who is assumed to be in the top rank position. Therefore, player 1 is regarded as a principal in our model.

We suppose that player 1 possesses the crucial asset. The choice of organizational form (structure) involves player 1 allowing players 2 and 3 to access the asset (or player 1). The three choices are joint ownership, direct access and indirect access. Therefore, our model is related to that of Rajan and Zingales (1998, 2001), who analyze differential access to an agent with a crucial asset as an organizational structure.[1]

We obtain the following results. If the investments of all members are (perfectly) complementary and essential for production, the horizontal organization is chosen. If investments of two players, including player 1, are essential and if the investment of another player is marketable, then a common agency arises in which the two players who make essential investments are both bosses. If two subordinates intend to invest their human capital in a pyramidal hierarchy, player 1 chooses this form. We examine a tier assignment problem in the vertical hierarchy when player 2 and player 3 are asymmetric. In our model, a player assigned to the bottom rank only invests if his or her investment is marketable. Because a player in the middle rank has bargaining power because of his or her position, he or she has a greater incentive to invest than does a player in the bottom rank. We show that if only one player's investment is marketable, the player who undertakes firm-specific investment is assigned to the middle rank. If the investments of both players are firm specific, the player whose investment increases the firm's value by more should be assigned to the middle rank. To complete the analysis, we compare the vertical hierarchy with the pyramidal hierarchy. The vertical hierarchy is only feasible when the owner can motivate a player who undertakes firm-specific investment by assigning him or her to the middle rank. However, the wage increase is large relative to the benefit when the

[1] This access is the ability to use, or work with, a critical resource.

degree of firm specificity is small. Then, the owner chooses the pyramidal hierarchy even though this does not persuade the player to invest.

Most closely related to our work is that of Hart and Moore (1990), who examine how the ownership of assets affects human capital investments; they also consider the boundary of the firm. To focus on the design of organizations, we do not consider the control structure for assets or the boundary of the firm. These are points of difference with Hart and Moore (1990).

There are two other points of difference with Hart and Moore (1990). First, expost bargaining in our model is based on a noncooperative approach. Hart and Moore adopted the Shapley value as a solution concept to the bargaining problem. Because they adopted a noncooperative approach to the action decision problem, their solution concept is inconsistent. Adopting the cooperative game approach is irrelevant because in some noncooperative bargaining game models, the Shapley value is represented as a Nash equilibrium outcome (see Gul, 1989, Hart and Mas-Colell, 1996, Hart and Moore, 1988 and Stole and Zwiebel, 1996). However, noncooperative bargaining games that implement the Shapley value feature players that have equal positions and are given equal treatment in the bargaining procedure.[2] Because the bargaining procedure of an organization may depend on its organizational structure, cooperative bargaining cannot reflect this. We consider intrafirm bargaining, given the organizational structure. The organizational structure affects the bargaining procedure and potential coalitional deviations in renegotiations. Therefore, bargaining power is represented by the bargaining procedure.

Second, Hart and Moore (1990) analyze the choice of the ownership and control structure of assets to maximize the social surplus. We consider a situation in which lump-sum transfers are not feasible ex ante and in which the Coase Theorem cannot be applied to our model. Therefore, in our paper, the organizational form is chosen to maximize the principal's payoff. This is because he or she has the right to select an organizational form. As Chandler (1962) stated, "structure follows strategy", we suppose that the principal organizes the firm in a way that is consistent with incentives and bargaining strategies.

Four noteworthy papers on organizational form are by Rajan and Zingales (2001), Demange (2004), Hart and Moore (2005) and Choe and Ishiguro (2005). Rajan and Zingales (2001) attempted to compare a vertical hierar-

[2]For example, each player is equally likely to be the proposer (as in Gul, 1989 and Hart and Mas-Colell, 1986). Alternatively, all players are lined up in random order with each ordering being equally likely, and each player makes a take-it-or-leave-it offer in that order (as in Hart and Moore, 1988). Alternatively, each player is equally able to renegotiate (as in Stole and Zwiebel, 1996).

chy and a horizontal hierarchy, focusing on the effects of specialization and competition. They showed that steep hierarchies increase physical capital investment and that flat hierarchies promote human capital investments in the organization. In their model, competition involves a player in the middle tier setting up a new company with his or her subordinates and competing with the original firm. Noting that many young, fast-growing firms are established by people who replicated or modified an idea encountered in their previous employment, they consider competition as one of the most important factors affecting organizational structure. Competition is formally represented by coalitional deviation in our model. Demange (2004) investigated organizational structure from the viewpoint of group stability and has shown that the hierarchical structure achieves efficient coordination and is not blocked by any subgroup consisting of a superior and his or her subordinates. She called such a subgroup a "team", and considered it as a unit of deviation that has some autonomy and can make decisions. However, she adopted a cooperative solution concept (the core), and ignored incentive problems associated with human capital investment.

In this paper, we apply the concept of coalitional deviation, as used by Rajan and Zingales (2001) and Demange (2004), to a noncooperative bargaining game in which an organization's return is allocated. The potential for coalitional deviation depends on the organizational form.

Hart and Moore (2005) and Choe and Ishiguro (2005) wrote papers on the allocation of authority in the organization. Hart and Moore (2005) regard the design of hierarchies as determining the decision-making authority and suppose that a hierarchy of authority over decisions can be contractually specified ex ante. They explain why coordinators should be senior to specialists and why pyramidal hierarchies may be optimal. The delegation of authority is ignored in our model. We focus instead on the relationship between organizational forms and incentives to undertake human capital investment. Choe and Ishiguro (2005) considered an organization that consists of a principal and two agents and that implements two project. They compare three types of organizational structure: centralization, under which the principal has all the decision-making authority; decentralization, under which the principal delegates authority to each agent; and (vertical) hierarchy, in which the principal determines the project undertaken by the direct subordinate and, in turn, the subordinate has authority over the project implemented by his or her subordinate. They showed that the optimal authority structure depends on the externalities (or coordination benefits) between the two projects and on the incentive to invest in human capital. Their three-player organization is similar to the one considered in our model. Choe and Ishiguro (2005) also considered who should be in the middle tier of the hierarchy when two agents are asymmetric in their abilities. However, expost bargaining is assumed to be bilateral, and they apply a symmetric Nash bargaining solution.

The rest of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we study the bargaining procedure in each organization. In Section 4, we examine the incentive problem and the choice of organizational form. Section 5 concludes the paper. Proofs of the theorems and propositions are in the Appendix.

2 The Model

We consider an organization consisting of three risk-neutral players. The set of players is denoted by $N = \{1, 2, 3\}$, and a coalition, S, of players is a subset of N. There is an asset $\{a\}$ that is essential for production. We assume that player 1 owns this asset. In other words, player 1 owns the organization. There are three periods, date 0, date 1 and date 2.

At date 0, an organizational form is selected by player 1. At date 1, each player $i \in N$ chooses their level of human capital investment $e_i \in \{0, 1\}$ noncooperatively. Human capital investment, e_i, enables player i to excel at certain tasks and affects the firm's value at date 2. For example, consider a computer software company organized by three people. Each of them engages in financing, marketing or programming. Human capital comprises the skills and knowledge that must be acquired by each person to develop the new software and sell it. If a person quits the firm, the firm cannot use his or her human capital. At date 2, players negotiate over the allocation of the return, and production occurs. At date 0, an organizational form is selected by player 1. At date 1, each player $i \in N$ chooses the level of human capital investment $e_i \in \{0, 1\}$.

We follow the incomplete contracting approach of Hart and Moore (1990). We suppose that production and the allocation of the return at date 2 cannot be included in a contract made at date 0 because of the complexities of investment and because of transaction costs. Hence, the initial contract specifies the organizational form only.

2.1 Organizational Forms

We study four kinds of organizational form: (i) the horizontal organization; (ii) the common agency; (iii) the pyramidal hierarchy; and (iv) the vertical hierarchy. There are three tiers in the organization. It is assumed that player 1, who is the owner of the firm, is in the first tier. At date 0, player 1 assigns players 2 and 3 to their tiers. Each organizational form is characterized by players' tier assignment. We introduce the tier assignment function $t : N \to \{1, 2, 3\}$. That is, $t(i) = k$ indicates that player i belongs to tier k. By assumption, $t(1) = 1$. A player in tier k is subordinate to a player in tier $k-1$, and a player in tier $k-1$ is superior to a player in tier k. Thus, each player's tier represents their rank in the organization. The triplet $(t(1), t(2), t(3))$ determines the form of the organization uniquely. We assume that if $t(i) = k$ (≥ 2) for some $i \in N$, then, for any tier $m < k - 1$, there exists a $j \in N$ such that $t(j) = m$. This implies that every player except for the one in tier 1 has a direct superior in the organization.

(i) Horizontal organization

We specify the *horizontal organization* as $(t(1), t(2), t(3)) = (1, 1, 1)$. All players belong to tier 1 and are on the same level as each other. Figure 1 represents the horizontal organization. Each circle indicates a player in the organization. The symbol '$\leftrightarrows$' between two players indicates that both players are in the same tier and are on an equal footing. We use $\{i \leftrightarrows j\}$ to indicate that players i and j belong to the same tier. The horizontal organization characterized by $(t(1), t(2), t(3)) = (1, 1, 1)$ is denoted by g^1.

(Figure 1)

(ii) Common agency

We refer to the organizational form characterized by either $(t(1), t(2), t(3)) = (1, 1, 2)$ or $(t(1), t(2), t(3)) = (1, 2, 1)$ as the *common agency*. Tier 1 consists of two players and tier 2 contains the remaining players. A player in tier 2 is subordinate to the two players in tier 1. The common agency, $(t(1), t(2), t(3)) = (1, 1, 2)$, is illustrated in Figure 2. The symbol '$\rightarrow$' between two players represents the relationship between the superior and the subordinate. In the figure, $\{(\text{player } i) \rightarrow (\text{player } j)\}$ indicates that player i is superior to player j. We denote the organizational form characterized by $(t(1), t(2), t(3)) = (1, 1, 2)$ (or by $(t(1), t(2), t(3)) = (1, 2, 1)$) as g^2 (or g^3).

(Figure 2)

(iii) Pyramidal hierarchy

The *pyramidal hierarchy* is represented by $(t(1), t(2), t(3)) = (1, 2, 2)$. Player 1 is in the top tier, and players 2 and 3 belong to the second tier. In this organization, player 1 is a direct superior to players 2 and 3, and players 2 and 3 are in the same position and have no subordinate. Figure 3 represents the pyramidal hierarchy, $(t(1), t(2), t(3)) = (1, 2, 2)$. The pyramidal hierarchy is denoted by g^4.

(Figure 3)

(iv) Vertical hierarchy

An organizational form such as $(t(1), t(2), t(3)) = (1, 2, 3)$ or $(t(1), t(2), t(3)) = (1, 3, 2)$ is a *vertical hierarchy*. In the vertical hierarchy, all players are totally ordered. The vertical hierarchy can be denoted by $\{1 \to 2 \to 3\}$ or by $\{1 \to 3 \to 2\}$. In the organizational form implied by $\{1 \to j \to k\}$, player 1 is in the top tier (tier 1), player j is in the second tier (tier 2) and player k is in the bottom tier (tier 3). Player 1 is a direct superior to player j, and player j is a direct superior to player k. Figure 4 represents the vertical hierarchies characterized by $(t(1), t(2), t(3)) = (1, 2, 3)$ and $(t(1), t(2), t(3)) = (1, 3, 2)$. We denote $(t(1), t(2), t(3)) = (1, 2, 3)$ by g^5 and denote $(t(1), t(2), t(3)) = (1, 3, 2)$ by g^6.

(Figure 4)

Denote by G^N the set of the organizational structures that include all the players. Thus, $G^N = \{g_1, g_2, \dots, g_6\}$. For player i and player j, the set $G^{\{i,j\}}$ consists of $\{i \to j\}$, $\{j \to i\}$ and $\{i \leftrightarrows j\}$. In addition, $G^{\{i\}} = \{\emptyset\}$. The generic element in G^S, $S \subseteq N$, is denoted by g^S.

2.2 Human Capital Investment

Each player chooses a level of human capital investment, $e_i \in \{0, 1\}$, at date 1. The cost of investment, e_i, to player i is represented by e_i. Each player has a binary investment choice. That is, if $e_i = 1$, player i makes an investment in human capital, but if $e_i = 0$, player i makes no investment. The triplet of investment levels of the players is denoted by $e = (e_1, e_2, e_3)$. The levels of e_1, e_2 and e_3 are observed by all players at the end of date 1.

2.3 The Bargaining Situations

At date 2, production occurs and the return is realized. Before production is conducted, there is bargaining over the return. We allow production by subcoalitions of N at date 2. Coalition $\{i, j\}$ between players i, j and the singleton coalition $\{i\}$ of player i could realize a return, but the feasible subcoalitions are limited by the organizational form. The possibility of production by subcoalitions affects the determination of the payoff allocation through negotiations. We assume that the selected organizational form and the levels of human capital investment are observable by all players. The returns that coalitions of players can achieve are commonly known by all players. Thus, a multilateral bargaining process is conducted under complete and symmetric information. Unlike Hart and Moore (1990), who used the Shapley value in a cooperative game approach, we adopt a noncooperative game approach to this bargaining problem.

The return of the organization depends on the member, S, the organizational structure, g^S, and the level of investment by the member, $e^S = (e_i)_{i \in S}$. The return is denoted by $v(g^S, S|e^S)$ for $S \subseteq N$.

However, if the return of the organization at date 2 depends on its organizational form, the choice of organizational form is affected by the expected return. [3] In order to focus on the relationship between the incentives to undertake human capital investment and the choice of organizational structure, we assume that revenue is the same in all types of organizational structure that contain the same members and have the same level of investment.

Assumption 1. The value of $v(g^S, S|e^S)$ does not depend on $g^S \in G^S$.

Let $v(S|e^S) = v(g^S, S|e^S)$.

Assumption 2. For any $e = (e_i, e_j, e_k)$, the function v satisfies the following conditions:

$$\begin{aligned} &v(N \mid (e_i, e_j, e_k)) \geq v(\{i,j\}|(e_i, e_j)) + v(\{k\}|e_k), \text{ and} \\ &v(\{i,j\}|(e_i, e_j)) \geq v(\{i\}|e_i) + v(\{j\}|e_j), \text{ for } i,j,k = 1,2,3, \end{aligned}$$

The above condition ensures the *superadditivity* of v.

Superadditivity means that if a coalition divides into partitions, the return achieved by the coalition is at least as great as the aggregate return achieved by the coalitions producing separately. Hart and Moore (1990) also assume superadditivity.

Assumption 3. For all $S \subseteq N$, $v(S \mid (e_i)_{i \in S})$ is increasing in e_i.

Assumption 3 implies that human capital investment by any member of the coalition enhances the return of the coalition. This means that each human capital investment is beneficial to the members of the organization.

We make the following assumption about the marginal return on investments.

[3] For example, Radner (1992) and Bolton and Dewatripont (1994) focused on the information process and communication costs. The organizational form is chosen to maximize the expected return, taking into account informational problems.

Assumption 4. For every $i, j = 1, 2, 3$, $i \neq j$, the following is satisfied:

$$\begin{aligned} v(N|(1, e^{N\setminus\{i\}})) &- v(N|(0, e^{N\setminus\{i\}})) \\ &\geq v(\{i,j\}|(1, e_j)) - v(\{i,j\}|(0, e_j)) \\ &\geq v(\{i\}|1) - v(\{i\}|0). \end{aligned}$$

Assumption 4 states that there are increasing returns to scale in investment. The marginal return on investment increases with the size of the coalition. This condition also implies that investments are complementary.

The net return for each coalition $N = \{1, 2, 3\}$, $\{i, j\}$ and $\{i\}$, where $i, j = 1, 2, 3$ and $i \neq j$, is defined by:

$$\begin{aligned} f(N|(e_1, e_2, e_3)) &= v(N|(e_1, e_2, e_3)) - \sum_{i=1}^{3} e_i, \\ f(\{i,j\}|(e_i, e_j)) &= v(\{i,j\}|(e_i, e_j)) - (e_i + e_j), \\ f(\{i\}|e_i) &= v(\{i\}|e_i) - e_i. \end{aligned}$$

We make the following assumption about the net return.

Assumption 5. The net return for coalition N is maximized when $(e_1, e_2, e_3) = (1, 1, 1)$. In other words, $f(N\,|(1, 1, 1)) \geq f(N \mid (e_1, e_2, e_3))$ for all $(e_1, e_2, e_3) \in \{0, 1\} \times \{0, 1\} \times \{0, 1\}$. Moreover, $f(N \mid (1, 1, 1)) > 0$.

Together with Assumption 2 and Assumption 5, it follows that the social surplus is maximized when all players make their human capital investments under a grand coalition N. All five assumptions are maintained throughout the paper.

2.4 Noncooperative Bargaining Games

The procedure for bargaining over the return at date 2 depends on the organizational form that is selected at date 0. The opportunity to propose how the return is allocated and the potential for coalitional deviations are different in each organizational form. Essentially, a player in a higher tier has more bargaining power in allocating the return than does one in a lower tier in the organizational form. For example, there is positional power over the command system in an army hierarchy. Furthermore, our specification is consistent with the notion of "structural holes" proposed by Burt (1992). A structural hole is a joint node that connects an individual or group to other groups. The individual who can span the hole has power. This is because the individual can control the flow of information and the availability of the crucial asset. An individual in a higher tier is closer to the crucial asset and has more bargaining power over bridging gaps. Rajan and Zingales's (1998, 2001) notion of access represents a similar bargaining power structure.

We assume that the ordering of proposers is determined by ranks in the hierarchy. A player in a higher tier can make a proposal before a player in a lower tier can. This bargaining procedure implies that player i is in a stronger position in negotiations than is player j. According to the principal agent model, a superior makes a take-it-or-leave-it offer to his or her subordinates. Thus, if player i is superior to player j, that is, if $i \rightarrow j$, then player i makes a take-it-or-leave-it offer to player j. Because there is no informational asymmetry between players in our model, player i extracts the entire net surplus from player j.

On the other hand, players in the same tier have the same bargaining power in negotiations over the return. To model this situation, we assume that players in the same tier have the same opportunity to propose a coalition and propose an allocation of the return. If players i and j are in tier 1 ($i \leftrightarrows j$), then players i and j each have a 50% chance of being selected as the proposer in the bargaining procedure. If either player rejects the proposal, negotiations go to the next round. A new randomly selected proposer is then chosen, and the process is repeated. There is no first-mover advantage among players in the same tier.

The possibility of coalitional deviations in the bargaining game also depends on the selected organizational form. Under a coalitional deviation, a coalition becomes independent of the existing organization and engages in market competition with the remaining members of the organization. It is assumed that a coalition consisting of players characterized by the relation '$\rightarrow$' can deviate from the organization in the bargaining process. Under coalitional deviation, if a superior decides to deviate, his or her subordinates have no choice but to follow their superior. This assumption is consistent with the assumption of competing teams made by Rajan and Zingales (2001). If a manager in tier k decides to compete in the n-tier vertical hierarchy modeled by Rajan and Zingales, $n-k$ subordinates follow the manager and produce together as a team. Demange (2004) also considered coalitional deviations to examine the stability of a hierarchical structure. In Demange's model, a team is considered as a unit of deviations, and a coalition T is a team if and only if, for every i and j in T, either i is superior to j, j is superior to i or a common superior exists in T to both j and i, and all players between the tier containing i and the tier containing j belong to T. Coalitional deviation in our model is consistent with the concept of blocking by teams in Demange's model. However, in our model, the stability requirements are stronger than those in Demange's model. In Demange's framework, in the two-tier pyramidal hierarchy, only a grand coalition, N, and singletons can deviate from the organization. By contrast, we allow deviations by coalitions consisting of player 1 and his or her subordinates.

We do not allow coalitional deviations by players characterized by the relation '$\leftrightarrows$'; that is, we do not consider the possibility of collusion by players in the same tier. Demange also excluded coalitional deviations by players in the same tier because she only allowed blocking by teams, and teams do not consist of players who are in the same tier. Moreover, if we allow collusion between players in the same tier, this may contradict the idea that a player in a higher rank has more bargaining power than does a player in a lower rank. For example, if player 2 can collude with player 3 in the pyramidal hierarchy, players 2 and 3 might have more bargaining power than does player 1. In the conclusion, we comment on the possibility that player 1 may do better if there is collusion between players 2 and 3.

The rules of bargaining procedures can be summarized by the following three principles.

1. Initially, a player in the highest tier can propose an allocation of the return. Then, a player in the second highest tier can propose. A superior player makes a take-it-or-leave-it offer to the subordinate.

2. If some players are in the same tier, each has an equal chance of being selected as the proposer.

3. A player can form a coalition with his or her subordinates (but a player cannot collude with a player in the same tier).

The bargaining procedure in each organizational form follows these three rules. For a formal description, see points (i)–(iv) below.

(i) The bargaining procedure in the horizontal organization

A noncooperative bargaining game in the horizontal organization at date 2 runs as follows. At every round $t = 1, 2, \ldots,$ one player is selected as a proposer, with equal probability, from among all players. The selected player i proposes either: (a) a coalition N and an allocation of the return $v(N \mid e)$ for the members of N; or (b) a singleton coalition $\{i\}$. In the latter case, the game terminates and the vector of returns (v_1, v_2, v_3) of $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ is realized; that is, player i gets the share $v(\{i\}|e_i)$. In the former case, all other players in N either accept or reject the proposal sequentially. If all other players in N accept the proposal, the agreed division of the return is enforced and the game ends. If some players reject the proposal, the bargaining goes on to the next round and a new proposer is randomly selected according to the same rules.

(ii) The bargaining procedure in the common agency

We focus on the organization g_2 in which players 1 and 2 belong to the top tier and player 3 is in the second tier. For the common agency g_3, we interchange player 2 with player 3 in the following bargaining game.

At every round $t = 1, 2, \ldots$, one player is selected as a proposer, with equal probability, from among the players in the first tier. The selected player $i \in \{1, 2\}$ proposes either: (a) a coalition N and a division of the return $v(N|(e_1, e_2, e_3))$; (b) a coalition $S = \{i, 3\}$ and a division of $v(\{i, 3\}|(e_i, e_3))$ for player i and player 3; or (c) a singleton coalition $\{i\}$. First, consider (a). If all the other players in N accept the proposal, it is agreed upon and enforced. The game ends. If some players reject the proposal, negotiations continue to the next round and a new proposer is randomly selected from the players in the first tier. Second, consider (b). Player i makes a take-it-or-leave-it offer of an allocation of the return $v(\{i, 3\}|(e_1, e_3))$ to player 3. If player 3 rejects the offer, the game ends with the allocation of the return $(v_1, v_2, v_3) = (v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$. If player 3 accepts the offer, it is enforced and the game terminates. Third, in case (c), the game ends with the vector of returns $(v_1, v_2, v_3) = (v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$.

(iii) The bargaining procedure in the pyramidal hierarchy

Player 1 is chosen as the proposer with certainty. Player 1 proposes either: (a) a coalition N and a division of the return $v(N|(e_1, e_2, e_3))$; (b) a coalition $S = \{1, j\}$, $j = 2, 3$, and an allocation of $v(\{1, j\}|(e_1, e_j))$ between player 1 and player j; or (c) a singleton coalition $\{1\}$. Under (a), player 1 makes a take-it-or-leave-it offer to players 2 and 3. If either player rejects the proposal, negotiations break down and the vector of returns $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ is realized. If players 2 and 3 both accept the offer, it is agreed upon and enforced. Under (b), player 1 makes a take-it-or-leave-it offer to player j. If player j accepts the proposal, it is enforced and the game ends. If player j rejects the proposal, at the breakdown of negotiations, an allocation of the return of $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ arises. When proposal (c) is made, the game ends with an allocation of the return of $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$.

(iv) The bargaining procedure in the vertical hierarchy

First, player 1 proposes either: (a) a coalition N; or (b) a singleton coalition $\{1\}$.[4] Under (b), the return allocation $(v_1, v_2, v_3) = (v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ is realized. Note that, under (a), player 1 makes a take-it-or-leave-it offer to

[4]The results of this paper are unaffected if player 1 has the additional options of forming a coalition only with the player in the middle tier or with the player in the bottom tier under Assumptions 1–5.

player 2 relating to the sum of player 2 and player 3's shares. If player 2 rejects the proposal, then player 2 chooses between: (i) a coalition $\{2,3\}$; and (ii) a singleton coalition $\{2\}$. Under (i), player 2 makes a take-it-or-leave-it offer to player 3 relating to the allocation of $v(\{2,3\}|(e_2,e_3))$. Then, player 3 either accepts or rejects the proposal. If player 3 accepts the offer, it is enforced. If player 3 rejects the offer, negotiations break down and the vector of returns is reduced to $(v_1,v_2,v_3) = (v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$. Under (ii), the return allocation $(v_1,v_2,v_3) = (v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ is realized and the game ends. If player 2 accepts the offer, then player 2 makes a take-it-or-leave-it offer to player 3 relating to the allocation of the total share proposed by player 1. If player 3 rejects the proposal, the allocation of the return is $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$. If player 3 accepts the proposal, this is agreed upon and enforced.

Each player's share of the return is as follows. When an allocation $(v_i)_{i\in N}$ of the return is agreed upon in round t, the return of player i is $\delta^{t-1}v_i$, where δ is a discount factor such that $0 \leq \delta < 1$.

We apply a stationary subgame perfect equilibrium (SSPE) as the solution concept to the noncooperative bargaining games at date 2. An SSPE is a subgame perfect equilibrium with the property that for every $t = 1, 2, \ldots$, the tth-round strategy of every player depends only on the set of all active players at round t. It is well known that in a noncooperative multilateral bargaining game (one with more than three players), there are multiple subgame perfect equilibria when the discount factor is close to unity. For this reason, the concept of an SSPE is almost invariably used for noncooperative multilateral bargaining models (see, for example, Chatterjee et al., 1993, Gul, 1989, Okada, 1996 and Ray and Vohra, 1999). Note that the stationarity of the solution concept only applies to the noncooperative bargaining games in the horizontal organization and the common agency; it is irrelevant to the noncooperative bargaining games in the pyramidal and vertical hierarchies.

Throughout the paper, we focus on the limit point of the SSPE of each bargaining game as δ approaches unity.

3 Equilibrium Strategies

Let us characterize the equilibrium strategies at each date. The solution concept that we apply to the whole game, which takes place from date 0 to date 2, is the subgame perfect equilibrium. The equilibrium strategies for the whole game can be obtained by the backward induction procedure. All proofs are presented in the Appendix.

3.1 Bargaining Outcomes at Date 2

First, let us consider the noncooperative bargaining games at date 2. The equilibrium bargaining strategies at date 2 under each organizational form are given in Theorems 1–7.

Horizontal Organization: When the horizontal organization is selected at date 0 and when the levels of investment for all players at date 1 are given by $e = (e_1, e_2, e_3)$, the equilibrium strategies in the bargaining game at date 2 are described by Theorems 1 and 2.

Theorem 1. *If the discount factor δ is close to unity and if the following condition is satisfied:*

$$\begin{aligned} &v(N|e)/3 \geq v(\{1\}|e_1), \textit{ and} \\ &v(N|e)/3 \geq v(\{2\}|e_2), \textit{ and} \\ &v(N|e)/3 \geq v(\{3\}|e_3), \end{aligned} \tag{1}$$

then, there exists an SSPE of the bargaining game in the horizontal organization. In the SSPE, every player $i = 1, 2, 3$ proposes a coalition N and an allocation of the return $(v_1, v_2, v_3) = (v(N|e)/3, v(N|e)/3, v(N|e)/3))$ at round 1. Moreover, the proposal is accepted in the SSPE.

Note that the return is divided between all players equally in the horizontal organization, which is independent of the contribution of each investment.

Theorem 2. *If the discount factor δ is close to unity and if condition (1) is not satisfied, then there is no SSPE of the bargaining game in the horizontal organization.*

Common Agency: For the common agency, we obtain the following three theorems.

Theorem 3. *If the discount factor δ is close to unity and if the following conditions are satisfied:*

$$\begin{aligned} &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{1,3\}|(e_1, e_3)) - v(\{3\}|e_3), \textit{ and} \\ &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{2,3\}|(e_2, e_3)) - v(\{3\}|e_3), \end{aligned} \tag{2}$$

then, there exists an SSPE of the bargaining game in the common agency.

In the SSPE, a proposer $i = 1, 2$ offers a coalition N and an allocation of the return $(v_1, v_2, v_3) = ((v(N|e) - v(\{3\}|e_3))/2, (v(N|e) - v(\{3\}|e_3))/2, v(\{3\}|e_3))$ at round 1. Moreover, the proposal is accepted in the SSPE.

Theorem 4. *If the discount factor δ is close to unity and if the following condition is satisfied:*

$$\begin{aligned} & v(\{1,3\}|(e_1,e_3)) \geq \\ & \qquad v(N|e) - \frac{1}{2}v(\{2,3\}|(e_2,e_3)) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3), \text{ and} \\ & v(\{2,3\}|(e_2,e_3)) \geq \\ & \qquad v(N|e) - \frac{1}{2}v(\{1,3\}|(e_2,e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{3\}|e_3), \end{aligned} \tag{3}$$

then, there exists an SSPE of the bargaining game in the common agency.

In the SSPE, player 1 proposes a coalition $\{1,3\}$ and a vector of returns of $(v_1, v_3) = (v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$ at round 1. At round 1, player 2 proposes a coalition $\{2,3\}$ and a vector of returns of $(v_2, v_3) = (v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$. Moreover, these proposals are accepted in the SSPE.

The expected equilibrium shares of the return in the above SSPE (Theorem 4) are given by:

$$\begin{aligned} v_1^* &= \frac{1}{2}\left(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{1\}|e_1), \\ v_2^* &= \frac{1}{2}\left(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{2\}|e_2), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

The following theorem shows the nonexistence of an SSPE in the common agency.

Theorem 5. *If the discount factor δ is close to unity and if conditions (2) and (3) are not satisfied, then there is no SSPE of the bargaining game in the common agency.*

Pyramidal Hierarchy: Next, we consider the pyramidal hierarchy. In this case, there always exists a subgame perfect equilibrium because the bargaining game is a finite-length extensive-form game with complete information. The stationarity of this equilibrium strategy is irrelevant. Moreover, each player's strategy in the subgame perfect equilibrium is uniquely determined for any δ when Assumptions 1–5 are satisfied.

Theorem 6. *There exists a subgame perfect equilibrium of the bargaining game in the pyramidal hierarchy. In the subgame perfect equilibrium, player 1 proposes a coalition N and an allocation of the return of $(v_1, v_2, v_3) = (v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3), v(\{2\}|e_2), v(\{3\}|e_3))$. Moreover, players 2 and 3 accept the proposal in the subgame perfect equilibrium.*

Vertical Hierarchy: Now consider the bargaining game in the vertical hierarchy. We obtain a unique subgame perfect equilibrium of the bargaining game in the vertical hierarchy.

Theorem 7. *There exists a subgame perfect equilibrium of the bargaining game in the vertical hierarchy g_5. In the subgame perfect equilibrium, player 1 proposes to player 2 a share in the return of $v(\{2,3\}|(e_2,e_3))$. Player 2 accepts this proposal and then proposes to player 3 a division of the share such that $(v_2,v_3) = (v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$. Player 3 accepts the proposal by player 2 in the subgame perfect equilibrium.*

In the above subgame perfect equilibrium, the return of each player is given by:

$$\begin{aligned} v_1^* &= v(N|e) - v(\{2,3\}|(e_2,e_3)), \\ v_2^* &= v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

It is easy to prove the same theorem in the case of the vertical hierarchy g_6 by replacing player 2 with player 3. In the subgame perfect equilibrium of the bargaining game in g_6, player 2 gets $v(\{2\}|e_2)$ and player 3 does $v(\{2,3\}|(e_2,e_3)) - v(\{2\}|e_2)$.

Remark. Remark. (Comparisons with the Shapley value) Hart and Moore (1990) adopt a cooperative game approach to the bargaining problem of allocating a return by applying the Shapley value as a solution concept. In our bargaining problem involving three players, the Shapley value of each player is given by

$$\begin{aligned} B_1(e) =& \frac{1}{3}(v(N|e) - v(\{2,3\}|(e_2,e_3))) + \frac{1}{6}(v(\{1,2\}|(e_1,e_2)) - v(\{2\}|e_2)) \\ &+ \frac{1}{6}(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)) + \frac{1}{3}v(\{1\}|e_1), \\ B_2(e) =& \frac{1}{3}(v(N|e) - v(\{1,3\}|(e_1,e_3))) + \frac{1}{6}(v(\{1,2\}|(e_1,e_2)) - v(\{1\}|e_1)) \\ &+ \frac{1}{6}(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)) + \frac{1}{3}v(\{2\}|e_2), \\ B_3(e) =& \frac{1}{3}(v(N|e) - v(\{1,2\}|(e_1,e_2))) + \frac{1}{6}(v(\{1,3\}|(e_1,e_3)) - v(\{1\}|e_1)) \\ &+ \frac{1}{6}(v(\{2,3\}|(e_2,e_3)) - v(\{2\}|e_2)) + \frac{1}{3}v(\{3\}|e_3). \end{aligned}$$

In our noncooperative bargaining games for the common agency, the pyramidal hierarchy and the vertical hierarchy, player k in the bottom tier gains only the stand-alone return $v(\{k\}|e_k)$ in equilibrium. Therefore, for all organizational forms except the horizontal hierarchy, the equilibrium return allocation differs from the Shapley value. If the three players are perfectly symmetrical in terms of the contributions to the return, in which case $v(\{1,2\}|(e_1,e_2)) = v(\{1,3\}|(e_1,e_3)) = v(\{2,3\}|(e_2,e_3))$ and $v(\{1\}|e_1) = v(\{2\}|e_2) = v(\{3\}|e_3)$, then the Shapley value reduces to the vector $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$. Thus, only if all players are perfectly symmetrical and identical does the Shapley value coincide with the equilibrium return vector for a noncooperative bargaining game in the horizontal organization. However, in general, these allocations differ.

3.2 Decisions about Human Capital Investment

At date 1, to maximize his or her expected payoff, each player decides whether to invest. The expected SSPE return for player i ($i = 1,2,3$) at date 2 is denoted by $v_i^*(e_1,e_2,e_3;g_j)$, which is determined according to the bargaining procedure at date 2 in organizational structure g_j. We denote by e_{-i} the combination of the human capital investments of all players except player i.

Definition 1. Given the organizational structure g_j, the vector $e^* = (e_1^*, e_2^*, e_3^*)$ is an *equilibrium pair of investments at date 1* if it satisfies, for all $i = 1,2,3$:

$$v_i^*(e_i^*, e_{-i}^*; g_j) - e_i^* \geq v_i^*(e_i, e_{-i}^*; g_j) - e_i \text{ for all } e_i \in \{0,1\}$$

According to the equilibrium strategies pursued at dates 1 and 2, player 1 selects an organizational form g_j to maximize his or her payoff.

4 Main Results

4.1 Results on the Organizational Structure

In this section, we examine what kind of organization is chosen in relation to the human capital investments of the players. Proofs of the propositions are in the Appendix.

Definition 2. The human capital investments e_1, e_2, e_3 are *perfectly complementary* if they satisfy the following conditions. For all $e = (e_1, e_2, e_3)$ containing $e_i = 0$:

$$v(N|e) = 0, \tag{4}$$

and, for all $S \subset N$ such that $S \neq N$ and for all $(e_i)_{i \in S}$,

$$v(S|(e_i)_{i \in S}) = 0. \tag{5}$$

Perfectly complementary investments imply that the human capital of three players generate no value unless they are used together. Condition (4) means that no return occurs at date 2 if any player does not make a human capital investment. Condition (5) implies that even if (sub)coalitions are formed, there would be no return in the coalition.

The following proposition characterizes situations in which the horizontal form is selected.

Proposition 1. *If the human capital investments e_1, e_2, e_3 are perfect complementary, the horizontal organization is chosen in equilibrium.*

It is optimal for player 1 to choose the horizontal form because the returns to players 2 and 3 are not sufficient to invest in another organizational form, and it is essential to induce players 2 and 3 to invest when human capital investments are perfectly complementary.

The horizontal organization corresponds to partnerships, such as those in accounting and law firms in the real world.[5] The knowledge and abilities of workers are the most important inputs in these industries. Human capital is not tradable. The perfect complementarity of human capital represents these properties. The disadvantage of partnerships relates to the free-rider problem. However, this problem does not arise when there is perfect complementarity.

Hart and Moore (1990) obtained a similar result. They showed that two assets should be owned or controlled jointly if they are only productive when used together. Proposition 1 states that player 1 should give players 2 and 3 the same level of authority. The hierarchy structures that have a boss and subordinates are not optimal when the investments of all members are complementary and essential to the firm.

Proposition 1 is also consistent with the traditional investigation of Williamson (1975). Williamson pointed out that collective organizations (nonhierarchical associations) may arise when indivisibilities of either physical assets or informational types are substantial and when learning-by-doing emerges predictably. The perfect complementarity of human capital investments implies that human capital investment by each player is essential to production and that the accumulated human capital of players is indivisible. Moreover, in our model, there is only one physical asset. Thus, the physical asset should be owned and utilized by the group to provide all players with the incentive to maximize joint profits.

Next, we consider the common agency. Many researchers have pointed out that an organization with two bosses is not desirable from the point of

[5] Milgrom and Roberts (1992) study partnerships in detail.

view of such factors as the information process (Bolton and Dewatripont, 2004, Radner, 1993), the delegation of authority under incomplete contracts (Hart and Moore, 2005) and group stability (Demange, 2004). However, in practice, one finds that subordinates are managed by two bosses. This organizational structure is known as a matrix structure. The common agency can be optimal in our model. We illustrate the conditions under which the common agency is selected in equilibrium.

Definition 3. A human capital investment e_i is *marketable* if player i has an incentive to make a human capital investment independently; that is, $v(\{i\}|1) - 1 \geq v(\{i\}|0) - 0$.

This condition implies that human capital is valuable in the market in its own right. When investment is general, human capital is equally valuable to other firms. Therefore, this condition is likely to be satisfied for general investments.

Proposition 2. *There exists an equilibrium in which the common agency is chosen and in which the efficient level of human capital investment* $e^* = (1, 1, 1)$ *is implemented.*

In the proof of Proposition 2, we provide an example in which the human capital investments of players 1 and 2, respectively, e_1 and e_2, are perfect complementary and in which the human capital investment of player 3, e_3, is marketable. Problems in the matrix structure involve conflict and coordination between bosses. In the example in Proposition 2, no value can be generated if any one of the bosses does not invest. Therefore, there is no conflict between bosses and the common agency can be optimal.

For example, player 1 concentrates on technical improvements and player 2 concentrates on management of the firm. Then, both types of human capital investment are needed by the firm. To provide player 2 with an incentive to invest, it is optimal for player 1 to give equal authority to player 2 and build a strong partnership. Honda, Google and Yahoo! are successful examples of the common agency.

In our model, a subcoalition (subgroup) can be formed, as implied by Theorem 4, if player 1 chooses the common agency at date 0. However, the next proposition states that an organization of subcoalitions is dominated by other organizational forms.

Proposition 3. *A subcoalition cannot be formed in equilibrium.*

Proposition 3 shows that division of the firm or the firm's boundaries does not matter in a game that incorporates superadditivity and excludes externalities. Player 1 cannot acquire any benefit from player 2 by forming a subcoalition $\{1, 3\}$. Because the return is assumed to be superadditive and increasing with scale, player 1 gets a larger payoff in the pyramidal hierarchy than in a subcoalition {1,3} even if player 2 does not invest.

In what follows, we assume that player 1 has already acquired human capital ($e_1 = 1$). Hence, we ignore the incentive problem of player 1.

Proposition 4. *If players 2 and 3 invest in human capital in the pyramidal hierarchy, player 1 chooses the pyramidal hierarchy at date 0.*

Player 1 can acquire the entire surplus in the pyramidal hierarchy. However, the player in the middle tier has some bargaining power in the vertical hierarchy. The common agency gives player 2 the same bargaining power as player 1 and the horizontal organization equal bargaining power to all members. Therefore, the optimal organizational form for player 1 is the pyramidal hierarchy if player 1 can persuade players 2 and 3 to invest.

4.2 Results on Hierarchies

Let us consider the optimal tier assignment in the vertical hierarchy. If players 2 and 3 are asymmetrical, which player should be assigned to the middle tier? The solution to this problem depends on the marketability and firm specificity of the players' investments.

When both investments are marketable, Proposition 4 implies that the vertical hierarchy is dominated by the pyramidal hierarchy from the viewpoint of player 1. Therefore, we consider two cases: in the first, only one investment is marketable; in the other, neither investment is marketable.

Suppose that the investment of player 2 is not marketable, but that of player 3 is marketable, as follows:

$$v(\{2\}|1) - v(\{2\}|0) < 1 \tag{6}$$

$$v(\{3\}|1) - v(\{3\}|0) \geq 1 \tag{7}$$

Proposition 5. *Organization g_5 dominates the organization g_6 given the satisfaction of conditions (6) and (7).*

Because of the incompleteness of contracts, the returns on the investments of the players depend on the marketability of their human capital investments and on the distribution of bargaining power. If investment is sufficiently general and valuable in the market, players have an incentive to invest voluntarily. However, if the investment is specific, the hold-up problem arises and player 1 can mitigate this problem to give player 2 a better bargaining position. The hierarchical structure, in which a player with a marketable investment is relegated to the bottom tier whereas one with a firm-specific investment is elevated to the middle tier, dominates the hierarchical structure, in which the players are assigned in the reverse order. For example, the skills in which computer programmers have invested are highly marketable, and these workers are typically assigned to the bottom rank.

Definition 4. A human capital investment e_i is *firm specific* if it is not marketable and satisfies the following condition:

$$v(\{1,i,j\}|(1,1,e_j)) - v(\{1,i,j\}|(1,0,e_j)) \\ > v(\{i,j\}|(1,e_j)) - v(\{i,j\}|(0,e_j)) \tag{8}$$

is satisfied.

Condition (8) is the same as Assumption 4 except that it contains an equality sign. Condition (8) implies that the marginal return of player i's investment is higher in the grand coalition than in the subcoalition that excludes player 1.[6] The greater is the difference between the left-hand side and the right-hand side of (8), the greater is the increase in e_i because of player 1's participation. We refer to the ratio of the left- and right-hand sides, $\Delta_i(e_j)$, as the degree of firm specificity.

$$\Delta_i(e_j) = \frac{v(\{1,i,j\}|(1,1,e_j)) - v(\{1,i,j\}|(1,0,e_j))}{v(\{i,j\}|(1,e_j)) - v(\{i,j\}|(0,e_j))}$$

When $\Delta_i(e_j)$ is large, the investment of player i is more specific in at the margin.

Proposition 6. *If the investments of players 2 and 3 are not marketable and if the investment of player 2 contributes more to the firm's return than does that of player 3 in the following sense:*

$$v(N|(1,1,0)) \geq v(N|(1,0,1)), \tag{9}$$
$$v(\{2,3\}|(1,0)) = v(\{2,3\}|(0,1)), \tag{10}$$

then, the organization g_5 dominates the organization g_6.

[6]The coalition that includes player 1 can utilize player 1's human capital and the physical asset. Therefore, this condition means that e_i is relation specific to e_1 and to the asset.

If neither investment is marketable, then the player whose investment contributes more to the firm's value should be assigned to the upper tier. By using the degree of firm specificity, conditions (9) and (10) imply that $\Delta_2(0) \geq \Delta_3(0)$. Proposition 6 suggests that if the human capital investment of player 2 is more firm specific than that of player 3, player 2 should be assigned to the middle tier.

Propositions 5 and 6 determine which agents the owner should assign to the middle tier in the hierarchical organization if the agents are asymmetric. Choe and Ishiguro (2006) also addressed this problem. Choe and Ishiguro showed that, if two agents have the same cost function, the agent whose project is more likely to succeed and whose marginal probability of human capital investment is higher should be in the middle tier. This is because this agent can be better motivated through empowerment. Their result is similar to ours in Proposition 6, although tier assignment in our model depends not only on the firm's revenue, but also on the market value of human capital investment.

Next, we compare the vertical hierarchy g_5 with the pyramidal hierarchy g_4. Proposition 4 states that if the investments of players 2 and 3 are marketable, player 1 prefers the pyramidal hierarchy to the vertical hierarchy. Proposition 5 implies that if one of the investments is marketable, the player with the marketable investment should be assigned to the bottom tier. Hence, it is sufficient to compare the pyramidal hierarchy with a vertical hierarchy when the investment of player 2, who is assigned to the middle tier, is not marketable. Proposition 7 deals with the case in which player 2 invests not in the pyramidal hierarchy, but in the vertical hierarchy ($v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)) \geq 1$). Proposition 8 deals with the case in which player 2 does not invest in either of the organizational types ($v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)) < 1$).

Proposition 7. *Assume that a player in the middle tier invests in the vertical hierarchy, but does not invest in the pyramidal hierarchy. The vertical hierarchy dominates the pyramidal hierarchy if and only if:*

$$\begin{aligned} &v(N|(1,1,e_3)) - v(N|(1,0,e_3)) \\ &\qquad \geq v(\{2,3\}|(1,e_3)) - v(\{2\}|0) - v(\{3\}|e_3), \end{aligned} \tag{11}$$

where $e_3 \in \{0,1\}$.

The intuition behind this result is straightforward. The benefit of the vertical hierarchy is that the owner can motivate player 2 to undertake human capital investment. The left-hand side of (11) represents the increase in the return on the investment of player 2. On the other hand, the vertical hierarchy gives player 2 some bargaining power over the expost return. The right-hand side of (11) represents the increase in the payment to player 2. Player 1 prefers the vertical hierarchy to the pyramidal hierarchy if and only if the investment of a player in the middle tier is not marketable and the benefit of the vertical hierarchy outweighs its cost.

Assumption 2 implies that $v(\{2,3\}|(1,e_3))-v(\{2\}|0)-v(\{3\}|e_3) \geq v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$. Assumption 4 implies that $v(N|(1,1,e_3))-v(N|(1,0,e_3)) \geq v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$. Given the value of $v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$, if $\Delta_i(e_j)$ is large, then $v(N|(1,1,e_3)) - v(N|(1,0,e_3))$ is large. Thus, (11) is satisfied when $\Delta_2(e_3)$ is sufficiently large. Therefore, Proposition 7 states that the vertical hierarchy is preferred to the pyramidal hierarchy if the investment of the player assigned to the middle tier is sufficiently firm specific. Our model suggests that a steeper hierarchy is adopted by organizations that require firm-specific human capital investment.

When $e_3 = 1$, under the vertical hierarchy, the efficient outcome, $e = (1,1,1)$, is implemented. However, when e_2 is not sufficiently specific to satisfy (11), player 1 prefers the pyramidal hierarchy even if he or she can motivate all subordinates to invest in the vertical hierarchy. Therefore, Proposition 7 shows that organizations in which incentives are weak are possible.

Proposition 8. *If a player in the middle tier does not invest in the vertical hierarchy, the vertical hierarchy is dominated by the pyramidal hierarchy.*

Propositions 7 and 8 imply that the vertical hierarchy can be optimal only if the owner can motivate players to undertake firm-specific investments by assigning them to the middle tier in the hierarchy. Steeper (vertical) hierarchies are more effective in inducing firm-specific human capital investments by players in the upper tier than are flatter ones.

Finally in this section, we compare the vertical hierarchy and the common agency from the point of view of player 2's incentives. Player 1 can motivate player 2 to invest in firm-specific human capital by choosing the vertical hierarchy and then assigning a subordinate to player 2. Alternatively, player 1 chooses the common agency and provides player 2 with an equal bargaining position. Is it better for player 1 to provide player 2 with a subordinate or an equal bargaining position? The answer depends on incentives and costs. If the below is satisfied, player 2 has a stronger incentive to invest in the common agency than in the vertical hierarchy:

$$\begin{aligned}&\frac{1}{2}\{v(N|(1,1,e_3)) - v(N|(1,0,e_3))\}\\ &\qquad \geq v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)).\end{aligned}$$

This condition holds when the degree of firm specificity is sufficiently high. However, if the degree of firm specificity is high, costs are higher in the common agency than in the vertical hierarchy. In the next subsection, we use a numerical example to show which organization is best.

4.3 Numerical Example

In this section, we specify the return and use numerical calculations to determine which organization is best. We assume that the return on investment is represented by, for $i, j = 1, 2, 3, i \neq j$:

$$v(N|(e_1, e_2, e_3)) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \beta_{12} e_1 e_2 + \beta_{23} e_2 e_3 + \beta_{31} e_3 e_1 + \gamma e_1 e_2 e_3,$$
$$v(\{i, j\}|(e_i, e_j)) = \alpha_i e_i + \alpha_j e_j + \beta_{ij} e_i e_j,$$
$$v(\{i\}|e_i) = \alpha_i e_i,$$

where α_i, β_{ij} and $\gamma \geq 0$.

Suppose that the parameters satisfy condition (2).[7] The condition can be expressed as:

$$\alpha_1 e_1 + \beta_{12} e_1 e_2 + \beta_{31} e_3 e_1 + \gamma e_1 e_2 e_3 \geq \alpha_2 e_2 + \beta_{23} e_2 e_3, \quad (12)$$
$$\alpha_2 e_2 + \beta_{12} e_1 e_2 + \beta_{23} e_2 e_3 + \gamma e_1 e_2 e_3 \geq \alpha_1 e_1 + \beta_{31} e_3 e_1. \quad (13)$$

These imply that the grand coalition N is formed when player 1 chooses a common agency.

In particular, when $\gamma > 0, \alpha_i, \beta_{ij} = 0$, the investments made by the three players are perfectly complementary. From Proposition 1, the horizontal organization is chosen. When $\alpha_1 = \alpha_2 = 0, \alpha_3 > 0$ and $\beta_{23} = \beta_{31} = 0$, Proposition 2 implies that the common agency is chosen. When $\alpha_2 \geq 1$ and $\alpha_3 \geq 0$, the investments of players 2 and 3 are both marketable. Proposition 4 implies that the pyramidal hierarchy is chosen.

In what follows, we consider in detail the case in which $\alpha_2 < 1$ and $\alpha_3 \geq 1$. In this case, player 3 chooses $e_3 = 1$ in the pyramidal hierarchy, the vertical hierarchy and the common agency. Player 2 does not invest in the pyramidal hierarchy. Player 2 invests in the vertical hierarchy if $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$. Player 2 invests in the common agency if $v(N|(1,1,1)) - v(N|(1,0,1)) \geq 2$. Therefore, which organization is optimal for player 1 depends on player 2's incentive and payment. The calculations are in the Appendix. Figure 5 shows how the optimal organizational form

[7] If these conditions are not satisfied, Proposition 3 implies that the common agency is not chosen.

varies with the degree of specificity of e_2. The region to the northwest of the 45-degree line satisfies the assumption of increasing returns to scale in investments. The figure shows that the horizontal hierarchy is not chosen in this case. Because (11) is satisfied for this specialization, the pyramidal hierarchy is chosen only when neither the vertical hierarchy nor the common agency can induce player 2 to invest. If player 2 invests in the vertical hierarchy, player 1 chooses this hierarchy whether or not $e_2 = 1$ is implemented in the common agency. Player 1 prefers the vertical hierarchy when he or she can induce player 2 to invest. This is because it costs more to give player 2 an equal bargaining position than it does to assign a subordinate to player 2. The common agency dominates other organizational forms only when player 2 invests in the common agency but does not invest in the vertical hierarchy. Note that the degree of specificity is higher at points in the figure that are further to the northwest. Thus, Figure 5 illustrates that the common agency is chosen when the investment of player 2 is sufficiently specific to deter player 2 from investing in the vertical hierarchy.

(Figure 5)

5 Conclusion

In this paper, we have examined how the choice of organizational forms depends on the characteristics of human capital investments. We compared four types of organization and showed that various organizational forms can emerge.

In our model, the pyramidal hierarchy is selected under very restrictive conditions. In the pyramidal hierarchy, it is particularly difficult to provide incentives for players 2 and 3 (the subordinates) to invest. However, the owner can use various instruments to provide incentives for employees in practice. For example, tournaments and relative performance payments can be used if subordinates engage in the same task.

Moreover, we excluded coalition formation among players of the same rank. However, player 1 (the owner of the organization) might be better off if there is collusion between players 2 and 3 in a pyramidal hierarchy. For example, suppose that players 2 and 3 are symmetric and that both undertake investments that are not marketable. Then, neither player would invest in a pyramidal hierarchy unless there were collusion. If players 2 and 3 can form a coalition in the pyramidal hierarchy, player 1 assigns $v(\{2,3\}|(e_2, e_3))$ to players 2 and 3, who share this equally. If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) > 2$, both players make human capital investments. Under the vertical hierarchy and the common agency, no more than one player can be persuaded to invest. For this reason, the pyramidal hierarchy with collusion may deliver the best outcome for player 1.

We assumed that there are no externalities between coalitions. However, if there were externalities, a subcoalition may arise in the pyramidal hierarchy when a player who does not invest is ejected from the organization. Then, the owner may improve incentives by choosing the pyramidal hierarchy. Maskin (2003) studies a bargaining model that incorporates an externality between coalitions and in which there is a subcoalition. Further research on externalities between coalitions is needed to clarify the design of organizations.

Appendix

Proofs of Theorems in Section 3

Proof of Theorem 1.

We provide the following two lemmas in order to prove Theorem 1. The lemmas hold for any discount factor δ.

Lemma 1. *In every SSPE of the bargaining game in the horizontal organization where the expected return vector of the players is (v_1, v_2, v_3) and each player i proposes a coalition S_i on the equilibrium plays, every player i proposes a solution (S_i, y^i) of the maximization problem:*

$$\max_{S,y}(v(S|(e_j)_{j\in S}) - \sum_{j\in S} y_j) \qquad \text{subject to } y_j \geq \delta v_j, \text{for all } j \in S, j \neq i. \tag{A1}$$

The proposal (S_i, y^i) is accepted in the SSPE.

Proof. Let $x^i = (x_1^i, x_2^i, x_3^i)$ be the expected equilibrium return vector when player i becomes the proposer at round 1. Because each player is selected as a proposer with probability 1/3 in the bargaining game under the horizontal organization, $v_i = \sum_{k=1}^3 x_i^k/3$ for $i = 1, 2, 3$. We denote m^i by the maximum value of (A1). We will prove $x_i^i = m^i$.

Let us start to prove $(x_i^i \leq m^i)$. Suppose that player i proposes $(S, \hat{y})$ such that $\hat{y}_i > m^i$. Note that S is either N or $\{i\}$ in the case of this bargaining game. Since m^i is the maximum value of (A1), $\hat{y}_j < \delta v_j$ for some $j \in S$ with $j \neq i$. It is optimal for j to reject i's proposal because j's continuation return is δv_j when he rejects the proposal. Then, the game goes on to round 2. As a result, player i obtains the discount payoff δv_i. It is follow from the superadditivity of v that $\sum_{j=1}^3 x_j^k \leq v(N|e)$ for all $k = 1, 2, 3$. Therefore, $v_1 + v_2 + v_3 \leq (\sum_{j=1}^3 \sum_{k=1}^3 x_j^k)/3 \leq v(N|e)$. Thus, the proposal with a coalition N and the return vector (v_1, v_2, v_3) is feasible. This implies that $v_i \leq m^i$. Because $\delta < 1$, we have $\delta v_i \leq v_i \leq m^i$. Player i gets only δv_i even if he demands a return greater than m^i. This proves $x_i^i \leq m^i$.

Next, let us prove ($x_i^i \geq m^i$). Any solution (S, y) of the problem (A1) satisfies $m^i = v(S|(e_j)_{j\in S}) - \sum_{j\in S, j\neq i} y_j$, where $y_j = \delta v_j$. For any $\varepsilon > 0$, define z such that

$$z_i = m^i - \varepsilon, \quad z_j = \delta v_j + \frac{\varepsilon}{|S|-1}.$$

If player i proposes (S, z), then it is accepted. Therefore, $x_i^i \geq z_i = m^i - \varepsilon$. By taking ε small enough, we can obtain $x_i^i \geq m^i$. Then, we have $x_i^i = m^i$.

Finally, since $\delta v_i < m^i$, player i proposes a coalition S_i and the return vector $(m^i, (\delta v_j)_{j\in S_i, j\neq i})$ at round 1. □

Lemma 2. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is* (v_1, v_2, v_3) *and player 1, 2 and 3 propose a coalition* N *on the plays of the equilibrium if and only if*

(i) for every i *such that* $i, j, k = 1, 2, 3$, $i \neq j \neq k$,

$$v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i). \tag{A2}$$

(ii) the expected return vector (v_1, v_2, v_3) *satisfies*

$$\begin{aligned} v_1 &= \frac{1}{3}(v(N|e) - \delta v_2 - \delta v_3) + \frac{2}{3}\delta v_1, \\ v_2 &= \frac{1}{3}(v(N|e) - \delta v_1 - \delta v_3) + \frac{2}{3}\delta v_2, \\ v_3 &= \frac{1}{3}(v(N|e) - \delta v_1 - \delta v_2) + \frac{2}{3}\delta v_3. \end{aligned} \tag{A3}$$

Proof. (only-if). In the SSPE, the expected return vector is (v_1, v_2, v_3) and all players propose the grand coalition N. Player i can propose either N or $\{i\}$ when he becomes a proposer. By appling Lemma 1 to the SSPE, we can obtain

$$v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \text{ for } i = 1, 2, 3, i \neq j \neq k.$$

Every player i proposes the return allocation $(x_j^i)_{j\in N}$ such that

$$x_i^i = v(N|e) - \delta v_j - \delta v_k, \quad x_j^i = \delta v_j, \quad x_k^i = \delta v_k.$$

This proposal is accepted at round 1. Therefore, by the definition of the bargaining game in the horizontal organization, the expected return vector (v_1, v_2, v_3) is given by (A3).

(if). Consider the strategy combination such that, player i proposes a coalition N and the return vector $(v(N|e) - \delta v_j - \delta v_k, \delta v_j, \delta v_k)$, and accepted any proposal y^i for player i if and only if $y^i \geq \delta v_i$. It is easy to see that the above strategy is a locally optimal choice for every player under condition (i) and (ii) in Lemma 2. □

Proof of Thoerem 1. By Lemma 2, the expected equilibrium return vector (v_1, v_2, v_3) which satisfies (A3) converges to $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$ as δ goes to 1. In addition, the condition (i) in Lemma 2 becomes

$$v(N|e)/3 \geq v(\{1\}|e_1), \ v(N|e)/3 \geq v(\{2\}|e_2), \ v(N|e)/3 \geq v(\{3\}|e_3).$$

These conditions are corresponding to (1) in Theorem 1. Lemma 2 (combining with Lemma 1) implies that in the SSPE, player 1, 2 and 3 all propose at round 1 a coalition N

and the return vector $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$ when δ is sufficiently close to one. The proposal is accepted in the SSPE.

Proof of Theorem 2.

We can prove the following lemmas about the existence of an SSPE in the same way as Lemma 2. We omit proofs of Lemma 3, 4 and 5.

Lemma 3. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i and j propose a coalition N and player k proposes a coalition $\{k\}$ on the plays of the equilibrium if and only if*

(i)

$$\begin{aligned}
&v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \quad \text{for } i \in N = \{1,2,3\}, \\
&v(N|e) - \delta v_i - \delta v_k \geq v(\{j\}|e_j) \quad \text{for } j \in N, \\
&v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j. \quad \text{for } k \in N.
\end{aligned}$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$\begin{aligned}
v_i &= \frac{1}{3}\left(v(N|e) - \delta v_j - \delta v_k\right) + \frac{1}{3}\delta v_i + \frac{1}{3}v(\{i\}|e_i), \\
v_j &= \frac{1}{3}\left(v(N|e) - \delta v_i - \delta v_k\right) + \frac{1}{3}\delta v_j + \frac{1}{3}v(\{j\}|e_j), \\
v_k &= \frac{1}{3}v(\{k\}|e_k) + \frac{2}{3}\delta v_k.
\end{aligned}$$

Lemma 4. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i proposes a coalition N and player j and k propose a coalition $\{j\}$ and a coalition $\{k\}$ on the plays of the equilibrium if and only if*

(i) for $i, j, k \in N = \{1,2,3\}$ with $i \neq j \neq k$,

$$\begin{aligned}
&v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \text{ and }, \\
&v(\{j\}|e_j) \geq v(N|e) - \delta v_i - \delta v_k \text{ and }, \\
&v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j.
\end{aligned}$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$\begin{aligned}
v_i &= \frac{1}{3}\left(v(N|e) - \delta v_j - \delta v_k\right) + \frac{2}{3}v(\{i\}|e_i), \\
v_j &= \frac{2}{3}v(\{j\}|e_j) + \frac{1}{3}\delta v_j, \\
v_k &= \frac{2}{3}v(\{k\}|e_k) + \frac{1}{3}\delta v_k.
\end{aligned}$$

Lemma 5. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i, j and k propose a singleton coalition $\{i\}$, $\{j\}$ and $\{k\}$ respectively on the plays of the equilibrium if and only if* *(i) for $i, j, k \in N = \{1, 2, 3\}$ with $i \neq j \neq k$,*

$$v(\{i\}|e_i) \geq v(N|e) - \delta v_j - \delta v_k \text{ and },$$
$$v(\{j\}|e_j) \geq v(N|e) - \delta v_i - \delta v_k \text{ and },$$
$$v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j.$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$v_i = v(\{i\}|e_i), \; v_j = v(\{j\}|e_j), v_k = v(\{k\}|e_k).$$

From condition (ii) in Lemma 3 (also, Lemma 4, Lemma 5), we can derive the expected return vector of the players (v_1^*, v_2^*, v_3^*) as δ goes to one. By substituting (v_1^*, v_2^*, v_3^*) for condition (i) in Lemma 3 (also, Lemma 4, Lemma 5), we can easily see that the condition (i) contradicts the superadditivity of v as $\delta \to 1$. This implies that there is no SSPE of the bargaining game in the horizontal organization when the discount factor is close to one. We complete the proof of Theorem 2.

Proof of Theorem 3.

In the common agency g_2, player 1 and 2 belong to tier 1 and have an equal opportunity (probability) to make a proposal in the bargaining game. We provide the following lemma. The lemma is proved in the same way as in Lemma 2. Therefore, we abbreviate the proof of Lemma 6

Lemma 6. *There exists an SSPE of the bargaining game in the common agency g_2 where the expected return vector of the players is (v_1, v_2, v_3) and player 1 and player 2 propose a coalition N on the plays of the equilibrium if and only if*

(i) for player 1,

$$v(N|e) - \delta v_2 - \delta v_3 \geq v(\{1, 3\}|(e_1, e_3)) - \delta v_3 \text{ and,}$$
$$v(N|e) - \delta v_2 - \delta v_3 \geq v(\{1\}|e_1),$$

and for player 2,

$$v(N|e) - \delta v_1 - \delta v_3 \geq v(\{2, 3\}|(e_2, e_3)) - \delta v_3 \text{ and,}$$
$$v(N|e) - \delta v_1 - \delta v_3 \geq v(\{2\}|e_2).$$

(ii) the expected return vector (v_1, v_2, v_3) satisfies

$$v_1 = \frac{1}{2}\left(v(N|e) - \delta v_2 - \delta v_3\right) + \frac{1}{2}\delta v_1,$$
$$v_2 = \frac{1}{2}\left(v(N|e) - \delta v_1 - \delta v_3\right) + \frac{1}{2}\delta v_2,$$
$$v_3 = v(\{3\}|e_3).$$

As in Lemma 1, it can be shown that in the above SSPE, player 1 proposes at round 1 a coalition N and the return vector $(v(N|e) - \delta v_2 - \delta v_3, \delta v_2, \delta v_3)$ for player 1, 2 and 3, and player 2 proposes at round 1 a coalition N and the return vector $(\delta v_1, v(N|e) - \delta v_1 - \delta v_3, \delta v_3)$. Moreover, these proposals always been accepted at round 1 in the SSPE.

If a discount factor δ goes to one, the expected return vector (v_1, v_2, v_3) in the SSPE converges to (v_1^*, v_2^*, v_3^*) such that

$$\begin{aligned} v_1^* &= \frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right), \\ v_2^* &= \frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

In addition, condition (i) in Lemma 6 is rewritten as

$$\begin{aligned} &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3), \\ &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{2,3\}|(e_1,e_3)) - v(\{3\}|e_3). \end{aligned}$$

Therefore, we can obtain Theorem 3 from Lemma 6 as $\delta \to 1$.

Proof of Theorem 4.

We can provide the following lemma. We omit the proof of Lemma 7 because it can be proved in the same way as in Lemma 2.

Lemma 7. *There exists an SSPE of the bargaining game in the common agency g_2 where the expected return vector of the players is (v_1, v_2, v_3) and player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition $\{2,3\}$ on the plays of the equilibrium if and only if*

(i) for player 1,

$$\begin{aligned} &v(\{1,3\}|(e_1,e_3)) - \delta v_3 \geq v(N|e) - \delta v_2 - \delta v_3 \text{ and,} \\ &v(\{1,3\}|(e_1,e_3)) - \delta v_3 \geq v(\{1\}|e_1), \end{aligned}$$

and for player 2,

$$\begin{aligned} &v(\{2,3\}|(e_2,e_3)) - \delta v_3 \geq v(N|e) - \delta v_1 - \delta v_3 \text{ and,} \\ &v(\{2,3\}|(e_2,e_3)) - \delta v_3 \geq v(\{2\}|e_2). \end{aligned}$$

(ii) the expected return vector (v_1, v_2, v_3) satisfies

$$\begin{aligned} v_1 &= \frac{1}{2}\left(v(\{1,3\}|(e_1,e_3)) - \delta v_3\right) + \frac{1}{2}\delta v(\{1\}|e_1), \\ v_2 &= \frac{1}{2}\left(v(\{2,3\}|(e_2,e_3)) - \delta v_3\right) + \frac{1}{2}\delta v(\{2\}|e_2), \\ v_3 &= v(\{3\}|e_3). \end{aligned}$$

If a discount factor δ goes to one, the expected return vector of the players in the SSPE converges to (v_1^*, v_2^*, v_3^*) such that

$$\begin{aligned} v_1^* &= \frac{1}{2}\left(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{1\}|e_1), \\ v_2^* &= \frac{1}{2}\left(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{2\}|e_2), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

Then, the condition (i) in Lemma 7 becomes

$$\begin{aligned}
&v(\{1,3\}|(e_1,e_3)) \geq \\
&\qquad v(N|e) - \frac{1}{2}v(\{2,3\}|(e_2,e_3)) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3), \text{ and} \\
&v(\{2,3\}|(e_2,e_3)) \geq \\
&\qquad v(N|e) - \frac{1}{2}v(\{1,3\}|(e_2,e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{3\}|e_3),
\end{aligned}$$

as δ is sufficiently close to one. This condition is same as (3) in Theorem 4. Thus, Lemma 7 implies Theorem 4 as $\delta \to 1$.

Proof of Theorem 5.

In order to prove Theorem 5, we follow the same procedures as the proof in Theorem 2. We must provide several lemmas about the existence of an SSPE of the bargaining game in the common agency. These lemmas gives a necessary and sufficient condition for the existence of an SSPE as in Lemma 2. In the bargaining game for the common agency g_2, the following SSPE should be considered except in Lemma 6 and 7: an SSPE in which (i)player 1 proposes a coalition N and player 2 proposes a coalition $\{2,3\}$ on the plays of the equilibrium, (ii)player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition N, (iii)player 1 proposes a coalition N and player 2 proposes a singleton coalition $\{2\}$, (iv)player 1 proposes a coalition $\{1\}$ and player 2 proposes a coalition N, (v)player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition $\{2\}$, (vi)player 1 proposes a coalition $\{1\}$ and player 2 proposes a coalition $\{2,3\}$, and (vii)player 1 and player 2 proposes a singleton coalition $\{1\}$ and $\{2\}$. Corresponding to each SSPE, the lemma is provided. Thus, seven lemmas would be provided. We does not describe these lemmas in full detail, and we also omit the proof of the lemmas.

We can see that each necessary and sufficient condition for the existence of the SSPE does not satisfied if a discount factor δ is sufficiently close to one under Assumption 1-5. Then, Theorem 5 is obtained.

Proof of Theorem 6.

We can determine a subgame perfect equilibrium of the bargaining game by backward induction procedures since the bargaining game in the pyramidal hierarchy is finite game with perfect information. Let us start with the response strategies for player 2 and 3 in tier 2 of the organization. If player 2 reject an offer from player 1, then negotiations break down and player 2 obtains the payoff of $v(\{2\}|e_2)$. Therefore, player 2 accepts a proposal y_2 such that $y_2 \geq v(\{2\}|e_2)$. Similarly, player 3 accepts a proposal y_3 such that $y_2 \geq v(\{3\}|e_3)$. Taking into accounts of the response of player 2 and 3, player 1 makes a take-it-or-leave-it offer of a division of the return $v(N|e)$ among the players. If player 1 offers $v(\{2\}|e_2)$ for player 2 and $v(\{3\}|e_3)$ for player 3, then he obtains the return of $(v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3))$. This return is the maximum return that player 1 can obtain in an acceptable offer. Furthermore, if player 1 makes any offer that is rejected, then he obtains at most $v(\{1\}|e_1)$, which is less than $v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3)$ by the superadditivity of v. Hence, player 1 proposes at round 1 a coalition N and an allocation of $(v_1^*, v_2^*, v_3^*) = (v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3), v(\{2\}|e_2), v(\{3\}|e_3))$. Moreover, the proposal is accepted at round 1.

Proof of Theorem 7.

The equilibrium strategies of each player are derived by the backward induction procedure. If player 3 rejects an offer by player 2, player 3 obtains the return of $v(\{3\}|e_3)$. Therefore, player 3 accepts an offer y_3 if and only if $y_3 \geq v(\{3\}|e_3)$. In the vertical hierarchy, player 2 has the alternative of deviating from the organization by coalition $\{2,3\}$. If player 2 does so, player 2 obtains $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$ and player 3 obtains $v(\{3\}|e_3)$. This implies that player 2 accepts a return y_2 such that $y_2 \geq v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$. Since $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3) \geq v(\{2\}|e_2)$ by superadditivity of v, player 2 does not reject the return $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$. Therefore, player 1 proposes a coalition N and offers $v(\{2,3\}|(e_2,e_3))$ as a share of player 2 and player 3. Then, player 2 accepts the proposal and offers $v(\{3\}|e_3)$ for player 3. This offer is also accepted. In the equilibrium, the expected return of each player is given by $(v_1^*, v_2^*, v_3^*) = (v(N|e) - v(\{2,3\}|(e_2,e_3)), v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$.

Proofs of Propositions in Section 4

Proof of Proposition 1.

By Assumption 5, we have

$$v(N|(1,1,1)) > \sum_{i=1}^{3} e_i = 3.$$

When the organizational structure is the horizontal organization, the allocation of return for player i ($i = 1,2,3$) in the SSPE is $v_i^* = v(N|e)/3$ by Theorem 1. Since $v(N|(1,1,1))/3 - 1 > v(N|(0,1,1))/3 = 0$, player 1 chooses $e_1 = 1$ at date 1. Since player 2 and player 3 face the same incentive problem, there exists an equilibrium that all players invest and a grand coalition is formed in the horizontal organization. The payoff of player 1 in the horizontal organization becomes $\pi_1^H = v(N|e)/3 - 1 > 0$.

Next, we consider the common agency. Because Theorem 3 holds when (e_1, e_2, e_3) is perfectly complementary, $v_3^* = 0$ by (5). Then, player 3 chooses $e_3 = 0$. This makes $v_1^* = v_2^* = 0$ and $e_1 = e_2 = e_3 = 0$. Hence, the payoff of player 1 in the common agency π_1^C is zero; $\pi_1^C = 0$.

In the pyramidal hierarchy, Theorem 6 implies that $v_2^* = v_3^* = 0$. Since player 2 and 3 make no investment, $(e_2 = e_3 = 0)$, it follows that $v(N|(e_1,0,0)) = 0$ and player 1 also does not invest; $e_1 = 0$. Hence, the payoff of player 1 in the pyramidal hierarchy is zero; $\pi_1^P = 0$. In the vertical hierarchy, we can obtain $\pi_1^V = 0$ by the same argument as in the pyramidal hierarchy, where π_1^V is the payoff of player 1 in the vertical hierarchy.

Therefore, it is optimal for player 1 to choose the horizontal organization.

Proof of Proposition 2.

We give an example that satisfies Assumption 1-5 and holds Proposition 2. Assume that $v(\{1\}|0) = v(\{2\}|0) = v(\{3\}|0)$.

We consider the case where only the investment of player 3 is marketable; $v(\{3\}|1) - v(\{3\}|0) \geq 1$, and investments of player 1 and player 2 are not marketable. In addition, it is assumed that

$$v(\{1\}|1) = v(\{1\}|0) = 0, \text{and } v(\{2\}|1) = v(\{2\}|0) = 0.$$

We assume that the firm's value is equal to $v(\{3\}|e_3)$ if player 1 and 2 do not invest. The additional return is generated only if the human capital investments of player 1 and player 2 would be together, that is,

$$v(N|(1,1,e_3)) > v(N|(1,0,e_3)) = v(N|(0,1,e_3)) = v(\{3\}|e_3), \tag{B1}$$
$$v(\{1,3\}|(e_1,e_3)) = v(\{2,3\}|(e_2,e_3)) = v(\{3\}|e_3). \tag{B2}$$

At first, we consider the incentive problem in the common agency g_2. Since the return of player 3 is $v(\{3\}|e_3)$, player 3 chooses $e_3 = 1$. Theorem 3 holds under conditions (B1) and (B2). Then, the proposer offers a coalition N in equilibrium. Let us consider the case in which the following conditions are satisfied:

$$\frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - \frac{1}{2}(v(N|(0,1,1)) - v(\{3\}|1)) \geq 1,$$
$$\frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - \frac{1}{2}(v(N|(1,0,1)) - v(\{3\}|1)) \geq 1.$$

In this case, player 1 and player 2 make their human capital investments. From (B1), both of conditions are reduced to $(v(N|(1,1,1)) - v(\{3\}|1))/2 \geq 1$. Then, the equilibrium payoff of player 1 when the common agency is selected is given by

$$\pi_1^C = \frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - 1 > 0. \tag{B3}$$

Since $v(\{2\}|1) = 0$, player 2 does not invest in the pyramidal hierarchy. Because $v(N|(e_1,0,1)) - v(\{2\}|0) - v(\{3\}|1) - e_1 = -e_1$, player 1 chooses $e_1 = 0$. Thus, the equilibrium payoff of player 1 if she chooses the pyramidal hierarchy is $\pi_1^P = v(N|(0,0,1)) - v(\{2\}|0) - v(\{3\}|1) = 0$. Therefore, player 1 prefers the common agency to the pyramidal hierarchy.

Next consider the incentive for player 2 in the vertical hierarchy. Note that $(v(\{2,3\}|(1,1)) - v(\{3\}|1)) - (v(\{2,3\}|(0,1)) - v(\{3\}|1)) = 0 < 1$. This implies that player 2 will choose $e_2 = 0$ at date 1. It is easy to see that player 1 also chooses $e_1 = 0$ if $e_2 = 0$. Thus, the equilibrium payoff of player 1 if she chooses the vertical hierarchy becomes $\pi_1^V = v(N|(0,0,1)) - v(\{2,3\}|(0,1)) = 0$, and, then, she prefers the common agency to the vertical hierarchy.

Finally, the incentive constraint of investment for player 1 in the horizontal organization is represented by

$$\frac{1}{3}v(N|(1,e_2,e_3)) - 1 \geq \frac{1}{3}v(N|(0,e_2,e_3)). \tag{B4}$$

If $e_2 = 0$, this condition (B4) is violated and player 1 does not invest, i.e., $e_1 = 0$. If $e_2 = 1$, the condition (B4) becomes

$$\frac{1}{3}\{v(N|(1,1,e_3) - v(3|e_3)\} \geq 1, \tag{B5}$$

where $e_3 \in \{0,1\}$. The payoff of player 1 in the case of $e = (1,1,1)$ is

$$\pi_1^H = \frac{1}{3}v(N|(1,1,1)) - 1. \tag{B6}$$

From (B3) and (B6), it follows that $\pi_1^C \geq \pi_1^H$ if $v(N|(1,1,e_3))/3 \geq v(3|e_3)$. If $v(N|(1,1,e_3))/3 < v(\{3\}|e_3)$, no SSPE exists in the horizontal organization by Theorem 2. The payoff of π_1^H in the case of $e = (1,1,0)$ under the horizontal organization is always smaller than that in the case of $e = (1,1,1)$.

There exists no SSPE in which $e = (0,0,1)$ is implemented because $v_1^* = v_2^* = v_3^* = v(N|(0,0,1))/3 < v(\{3\}|1)$. When $e = (0,0,0)$, $\pi_1^H = 0$. Therefore, the horizontal organization is dominated by the common agency with $e = (1,1,1)$.

Proof of Proposition 3.

We shall show that the organization of a subcoalition in Theorem 4 is dominated by the pyramidal hierarchy.

From Theorem 4 and Theorem 6, we can see that the incentive problem for player 3 is same in both the common agency and the pyramidal hierarchy. Player 1 invests in the common agency if

$$\frac{1}{2}v(\{1,3\}|(1,e_3)) - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{1\}|1) - \frac{1}{2}v(\{1\}|0) \geq 1. \tag{B7}$$

In the pyramidal hierarchy, player 1 invests if

$$v(N|(1,e_2,e_3)) - v(N|(0,e_2,e_3)) \geq 1. \tag{B8}$$

Assumption 4 implies that the left-hand side of (B8) is larger than that of (B7). Then, there are three cases in which (i)player 1 invest in both organizations, (ii)player 1 does not invest in both organizations and (iii)player 1 invests in the pyramidal hierarchy but not in the common agency. In the cases of (i) and (ii), the level of e_1 is same in both organizations. When player 1 chooses the same investment level, we can obtain that

$$\begin{aligned}
\pi_1^P - \pi_1^C &= v(N|(e_1,e_2,e_3)) - v(\{2\}|e_2) - v(\{3\}|e_3) \\
&\qquad - \frac{1}{2}v(\{1,3\}|(e_1,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|e_1) \\
&\geq \frac{1}{2}v(N|(e_1,e_2,e_3)) + \frac{1}{2}v(\{1,3\}|(e_1,e_3)) + \frac{1}{2}v(\{2\}|e_2) \\
&\qquad - v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1,3\}|(e_1,e_3)) - \frac{1}{2}v(\{1\}|e_1) \\
&= \frac{1}{2}v(N|(e_1,e_2,e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) \geq 0
\end{aligned}$$

This implies that the payoff of player 1 in the pyramidal hierarchy is greater than that in the common agency. In the case of $e_1 = 0$ in the common agency and $e_1 = 1$ in the pyramidal hierarchy, we have

$$\begin{aligned}
\pi_1^P - \pi_1^C &= v(N|(1,e_2,e_3)) - v(\{2\}|e_2) - v(\{3\}|e_3) - 1 \\
&\qquad - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0) \\
&\geq v(N|(0,e_2,e_3)) + 1 - v(\{2\}|e_2) - v(\{3\}|e_3) - 1 \\
&\qquad - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0) \\
&\geq \frac{1}{2}v(N|(0,e_2,e_3)) + \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{2\}|e_2) - v(\{2\}|e_2) \\
&\qquad - v(\{3\}|e_3) - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0) \\
&= \frac{1}{2}v(N|(0,e_2,e_3)) - \frac{1}{2}v(\{1\}|0) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) \geq 0.
\end{aligned}$$

This means that the payoff in the pyramidal hierarchy is greater than that in the common agency. Therefore, the payoff for player 1 in forming a subcoalition under the common agency is always smaller than that in the pyramidal hierarchy.

Proof of Proposition 4.

According to Theorem 6, if the pyramidal hierarchy is chosen, the equilibrium return at date 2, given $e = (e_1, e_2, e_3)$, is represented by

$$\begin{aligned} v_1^*(e) &= v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3), \\ v_2^*(e) &= v(\{2\}|e_2), \\ v_3^*(e) &= v(\{3\}|e_3). \end{aligned}$$

Since both of player 2 and player 3 will choose to invest, the following conditions must be satisfied:

$$v(\{2\}|1) - v(\{2\}|0) \geq 1, \tag{B9}$$

$$v(\{3\}|1) - v(\{3\}|0) \geq 1. \tag{B10}$$

The equilibrium payoff of player 1 in the pyramidal hierarchy is given by

$$\pi_1^P(1,1,1) = v(N|(1,1,1)) - v(\{2\}|1) - v(\{3\}|1). \tag{B11}$$

Next we consider the vertical hierarchy. From (B10), player 3 chooses $e_3 = 1$. Since $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$ (by Assumption 4), player 2 chooses $e_2 = 1$. Therefore, the equilibrium payoff of player 1 in the vertical hierarchy is given by

$$\pi_1^V(1,1,1) = v(N|(1,1,1) - v(\{2,3\}|(1,1)). \tag{B12}$$

By Assumption 2, we can obtain that $\pi_1^P(1,1,1) \geq \pi_1^V(1,1,1)$.

Let us consider the horizontal organization. Since there is an SSPE in the horizontal organization only if $v(N|e)/3 \geq v(\{i\}|e_1)$ for $i = 1,2,3$, it is enough to restrict to such the case. Player 1 can get the maximum payoff at $(e_1, e_2, e_3) = (1,1,1)$. That is,

$$\pi_1^H(1,1,1) = v(N|(1,1,1))/3. \tag{B13}$$

in the horizontal organization. Using the condition that $v(N|e)/3 \geq v(\{i\}|e_i)$, $i = 1,2,3$, we can obtain that $\pi_1^P(1,1,1) \geq \pi_1^H(1,1,1)$.

Finally, we compare the common agency with the pyramidal hierarchy. From Proposition 3, it is sufficient to show that the common agency in the case of Theorem 3 is dominated by the pyramidal hierarchy. Since the incentive problem of player 3 is same in both organizational forms, player 3 chooses $e_3 = 1$ in the common agency. The payoff of player 1 is maximized at $e = (1,1,1)$ in the common agency. That is,

$$\pi_1^C(1,1,1) = \frac{1}{2}\left\{v(N|(1,1,1)) - v(\{3\}|1)\right\}. \tag{B14}$$

From the condition in Theorem 3, it follows that $v(N|(1,1,1))/2 \geq v(\{2,3\}|(1,1) - v(\{3\}|1)/2$. This implies that $\pi_1^P(1,1,1) \geq \pi_1^C(1,1,1)$. Therefore, if $(e_2, e_3) = (1,1)$ can be implemented under the pyramidal hierarchy, then player 1 chooses the pyramidal hierarchy at date 2.

Proof of Proposition 5.

If conditions (6) and (7) are satisfied, $e_3 = 1$ in g_5 and $e_2 = 0$ in g_6.

If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$, then, $e_2 = 1$ in g_5. Then, the payoff of player 1 in g_5 is given by

$$v(N|(1,1,1)) - v(\{2,3\}|(1,1)). \tag{B15}$$

The payoff of player 1 in g_6 is

$$v(N|(1,0,e_3)) - v(\{2,3\}|(0,e_3)), \tag{B16}$$

where $e_3 \in \{0,1\}$. Then, by Assumption 5, we obtain that $(B15) \geq (B16)$.

If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) < 1$, then $e_2 = 0$ in g_5. Therefore, the payoff of player 1 in g_5 is

$$v(N|(1,0,1)) - v(\{2,3\}|(0,1)). \tag{B17}$$

The payoff of player 1 in g_6 is given by (B16). If $e_3 = 1$ in g_6, then, we have that $(B17) = (B16)$. If $e_3 = 0$ in g_6, $(B17) \geq (B16)$ by Assumption 5. Hence, g_5 dominates g_6.

Proof of Proposition 6.

Since e_2 and e_3 are not marketable, a player in the bottom tier does not invest. Thus, $e_3 = 0$ in g_5 and $e_2 = 0$ in g_6.

Under Condition (10), a player in the middle tier of g_5 and g_6 has a same incentive to invest the human capital. If a player in the middle tier invest in either g_5 and g_6, the payoff of player 1 in g_5 is

$$v(N|(1,1,0)) - v(\{2,3\}|(1,0)), \tag{B18}$$

and that in g_6 is given by

$$v(N|(1,0,1)) - v(\{2,3\}|(0,1)). \tag{B19}$$

From (9) and (10), it follows that (B18) $\geq$ (B19).

If a player in the middle tier does not invest in either g_5 and g_6, the payoff of player 1 is same in g_5 and g_6 and that is given by $v(N|(1,0,0)) - v(\{2,3\}|(0,0))$. Hence, player 1 prefers g_5 to g_6.

Proof of Proposition 7.

If the investment of player 3 is marketable but that of player 2 is not marketable, Proposition 5 implies the player 2 is superior to player 3 in equilibrium. Since the pair of $e_2 = 0$ and $e_3 = 1$ is implemented in the pyramidal hierarchy, the payoff of player 1 is given by

$$\pi_1^P = v(N|(1,0,1)) - v(2|0) - v(3|1).$$

Since $e_2 = e_3 = 1$ in the vertical hierarchy, the payoff of player 1 is

$$\pi_1^V = v(N|(1,1,1)) - v(\{2,3\}|(1,1)).$$

Hence, $\pi_1^V \geq \pi_1^P$ if and only if

$$v(N|(1,1,1)) - v(N|(1,0,1)) \geq v(\{2,3\}|(1,1)) - v(\{2\}|0) - v(\{3\}|1),$$

When both of investments are not marketable, the payoff of player 1 from the pyramidal hierarchy is represented by

$$\pi_1^P = v(N|(1,0,0)) - v(2|0) - v(3|0),$$

because $e_2 = e_3 = 0$ in equilibrium. Since $e_2 = 1$ and $e_3 = 0$ are implemented in the vertical hierarchy, the payoff of player 1 is given by

$$\pi_1^V = v(N|(1,1,0)) - v(\{2,3\}|(1,0)).$$

Therefore, $\pi_1^V \geq \pi_1^P$ if and only if

$$v(N|(1,1,0)) - v(N|(1,0,0)) \geq v(\{2,3\}|(1,0)) - v(\{2\}|0) - v(\{3\}|0).$$

This completes the proof.

Proof of Proposition 8.

First, if both of the investments are marketable, Proposition 5 implies that the vertical is dominated by the pyramidal hierarchy.

Next, if the investment of player 2 is not marketable and that of player 3 is marketable, the payoff of player 1 is given by

$$\pi_1^P = v(N|(1,0,1)) - v(2|0) - v(3|1)$$

in the pyramidal hierarchy, and that in the vertical hierarchy is

$$\pi_1^V = v(N|(1,0,1)) - v(\{2,3\}|(0,1)).$$

By Assumption 2, we can obtain that $\pi_1^P \geq \pi_1^V$.

Finally, if both of investments are not marketable, the payoff of player 1 is

$$\pi_1^P = v(N|(1,0,0)) - v(2|0) - v(3|0)$$

in the pyramidal hierarchy and that in the vertical hierarchy g_5 is given by

$$\pi_1^V = v(N|(1,0,0)) - v(\{2,3\}|(0,0)).$$

By Assumption 2, we have $\pi_1^P \geq \pi_1^V$. Hence, the vertical hierarchy in which a player in the middle tier does not invest is dominated by the pyramidal hierarchy.

Caluculations in Section 4.3

If $\alpha_2 < 1$, $\alpha_3 \geq 1$, player 3 chooses $e_3 = 1$ in the pyramidal, vertical hierarchy and common agency. Since $e_2 = 0$ in the pyramidal hierarchy, the payoff of player 1 is

$$\pi^P(1,0,1) = \alpha_1 + \beta_{31}. \tag{C1}$$

If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$, i.e., $\alpha_2 + \beta_{23} \geq 1$, player 2 chooses $e_2 = 1$ in the vertical hierarchy. If $v(N|(1,1,1)) - v(N|(1,0,1)) \geq 2$, i.e., $\alpha_2 + \beta_{12} + \beta_{23} + \gamma \geq 2$, player 2 chooses $e_2 = 1$ in the common agency. Thus there are four cases.

Case 1. ($\alpha_2 + \beta_{23} \geq 1$ and $\alpha_2 + \beta_{12} + \beta_{2,31} + \gamma \geq 2$)

Player 2 chooses to invest in both organizations. Thus,

$$\pi^V(1,1,1) = \alpha_1 + \beta_{12} + \beta_{31} + \gamma \tag{C2}$$

$$\pi^C(1,1,1) = \frac{1}{2}(\alpha_1 + \alpha_2 + \beta_{12} + \beta_{23} + \beta_{31} + \gamma). \tag{C3}$$

It follows from (C1) and (C2) that $\pi^V(1,1,1) > \pi^P(1,0,1)$. To form grand coalition in the common agency, condition (12) must hold, but (12) implies that $\pi^V(1,1,1) > \pi^C(1,1,1)$. In the horizontal organization, it is ambiguous where player 2 and 3 invest or not. When $e_2 = 1$ and $e_3 = 1$, player 1 gets the largest payoff in the horizontal organization. That is

$$\pi^H(1,1,1) = \frac{1}{3}(\alpha_1 + \alpha_2 + \alpha_3 + \beta_{12} + \beta_{23} + \beta_{31} + \gamma). \tag{C4}$$

By Theorem 2, one can obtain that $\pi^C(1,1,1) > \pi^H(1,1,1)$. Hence, the vertical hierarchy is chosen in this case.

Case 2. ($\alpha_2 + \beta_{23} < 1$ and $\alpha_2 + \beta_{12} + \beta_{2,31} + \gamma \geq 2$)
Player 2 chooses to invest in the common agency but not in the vertical hierarchy. Proposition 8 implies that the vertical hierarchy is never chosen ($\pi^P(1,0,1) > \pi^V(1,0,1)$). Since the payoff in the common agency and the horizontal organization is same as in the first case, we have $\pi^C(1,1,1) > \pi^H(1,1,1)$. (13) implies that $\pi^C(1,1,1) > \pi^P(1,0,1)$. Hence, the common agency is chosen in this case.

Case 3. ($\alpha_2 + \beta_{23} \geq 1$ and $\alpha_2 + \beta_{12} + \beta_{23} + \gamma < 2$)
Player 2 chooses to invest in the vertical hierarchy but not in the common agency. From (C1) and (C2), we have $\pi^V(1,1,1) > \pi^P(1,0,1)$. Since $\pi^C(1,0,1) = (1/2)(\alpha_1 + \beta_{31})$, we can obtain that $\pi^V(1,1,1) > \pi^C(1,0,1)$. Player 2 chooses to invest in the horizontal organization iff $\alpha_2 + \beta_{12} + \beta_{23}e_3 + \gamma e_3 < 3$, but this condition does not be satisfied in this case. Thus, $e_2 = 0$ in the horizontal organization. Player 3 chooses to invest iff $(1/3)(\alpha_3 + \beta_{31}) \geq 1$. If this condition does not satisfied and $e_3 = 0$, we have $\pi^H(1,0,0) = (1/3)\alpha_1 < v(1|1)$. Then, Theorem 2 implies that there is no SSPE in the horizontal organization. Therefore, if there is an SSPE in the horizontal organization, $(1/3)(\alpha_3 + \beta_{31}) \geq 1$ and $\pi^H(1,0,1) > v(i|1)$ must be hold. But we can show that $\pi^V(1,1,1) > \pi^H(1,0,1)$ by these conditions. Hence, the vertical hierarchy is chosen in this case.

Case 4. ($\alpha_2 + \beta_{23} < 1$ and $\alpha_2 + \beta_{12} + \beta_{23} + \gamma < 2$)
Player 2 doesnot invest in both organizational forms. We can get that $\pi_P(1,0,1) \geq \pi^V(1,0,1)$ and $\pi^P(1,0,1) > \pi^C(1,0,1)$ immediately. By the same argument as in Case 3, the condition which requires for an existence of an SSPE in the horizontal organization impies that $\pi^P(1,0,1) > \pi^H(1,0,1)$. Therefore the pyramidal hierarchy is chosen in this case.

References

[1] Bolton, Patrick. and Mathias Dewatripont. 1994, "The Firm as a Communication Network." *Quarterly Journal of Economics* **109** (November): 809-839.

[2] Burt, Ronald. S. 1992, *Structural Holes: The Social Structure of Competition*, Cambridge, MA: Harverd University Press.

[3] Chandler, Alfred. D. 1962, *Stategy and Structure*, Cambridge, MA: MIT Press.

[4] Chatterjee, Kalyan, Bhaskar Dutta, Debraj Ray, and Kunal Sengupta. 1993, "A Noncooperative Theory of Coalitional Bargaining." *Review of Economic Studies* **60**: 463-477.

[5] Choe, Chongwoo. and Shingo Ishiguro. 2005, "Why Are There Hierarchies? An Incomplete Contracting Approach."

[6] Demange, Gabrielle. 2004, "On Group Stability in Hierarchies and Networks." *Journal of Political Economy* **112**: 754-778.

[7] French, John R. P. Jr. and Bertram Raven. 1959, "The Bases of Social Power." in Cartwright, Dorwin (ed.), *Studies in Social Power*, Ann Arbor: University of Michigan, Institute for Research.

[8] Gul, Faruk. 1989, "Bargaining Foundations of Shapley Value." *Econometrica* **57** (January): 81-95.

[9] Hart, Oliver and John Moore. 1988, "Incomplete Contracts and Renegotiation." *Econometrica* **56** (July), 755-785.

[10] Hart, Oliver and John Moore. 1990, "Property Rights and the Nature of the Firm." *Journal of Political Economy* **98**(December): 1119-1158.

[11] Hart, Oliver and John Moore. 2005, "On the Design of Hierarchies: Coordination versus Specialization." *Journal of Political Economy* **113** (August): 675-702.

[12] Hart, Sergiu and Andrew Mas-Colell. 1996, "Bargaining and Value." *Econometrica* **64** (March): 357-380.

[13] Itoh, Hideshi. 1992, "Cooperation in Hierarchical Organizations: A Incentive Perspective." *Journal of Law, Economics, and Organization* **8**: 321-345.

[14] Maskin, Eric S. 2003, "Bargaining, Coalitions, and Externalities." mimeo, Institute for Advanced Studies.

[15] Milgrom, Paul and John Roberts. 1992, *Economics, Organization and Management*, Englewood Cliffs, NJ: Prentice Hall.

[16] Okada, Akira. 1996, "A Noncooperative Coalitional Bargaining Game with Random Proposers." *Games and Economic Behavior* **16** (September): 97-108.

[17] Radner, Roy. 1993, "The Organization of Decentralized Information Processing." *Econometrica* **61** (September): 1109-1146.

[18] Rajan, Raghuram G., and Julie Wulf. 2003, "The Flattening Firm: Evidence from Panel Data on the Changing Nature of Corporate Hierarchies. " Working Paper, GSB, University of Chicago.

[19] Rajan, Raghuram G., and Luigi Zingales. 2001, "The Firm as a Dedicated Hierarchy: A Theory of the Origin and Growth of Firms." *Quarterly Journal of Economics* **116** (August): 805-851.

[20] Ray, Debraj, and Rajiv Vohra. 1999, "A Theory of Endogenous Coalition Structures." *Games and Economic Behavior* **26**: 286-336.

[21] Stole, Lars A., and Jeffrey Zwiebel. 1996, "Intra-firm Bargaining under Non-binding Contracts." *Review of Economic Studies* **63**: 375-410.

[22] Stole, Lars A., and Jeffrey Zwiebel. 1996, "Organization Design and Technology Choice under Intrafirm Bargaining." *American Economic Review* **86** (March): 195-222.

[23] Williamson, Oliver E., 1975, *Markets and Hierarchies: Analysis and Antitrust Implications*, The Free Press, New York.

Figure 1 Horizontal Organization

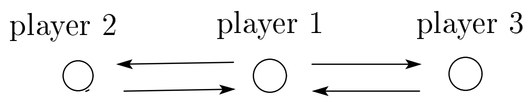


Figure 2 Common Agency

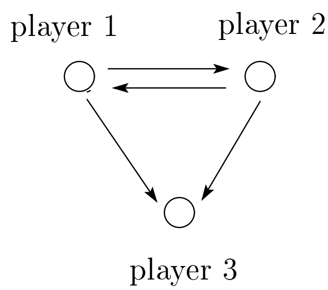


Figure 3 Pyramidal Hierarchy

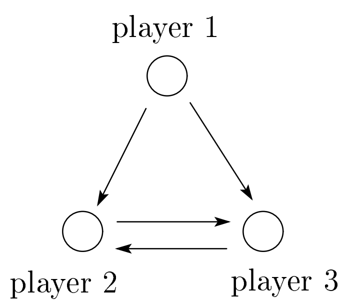


Figure 4 Vertical Hierarchy

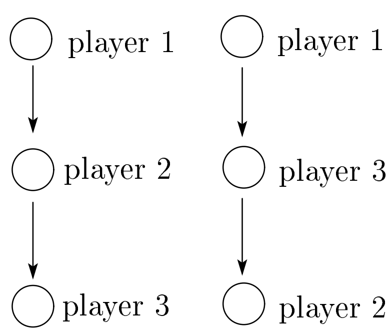
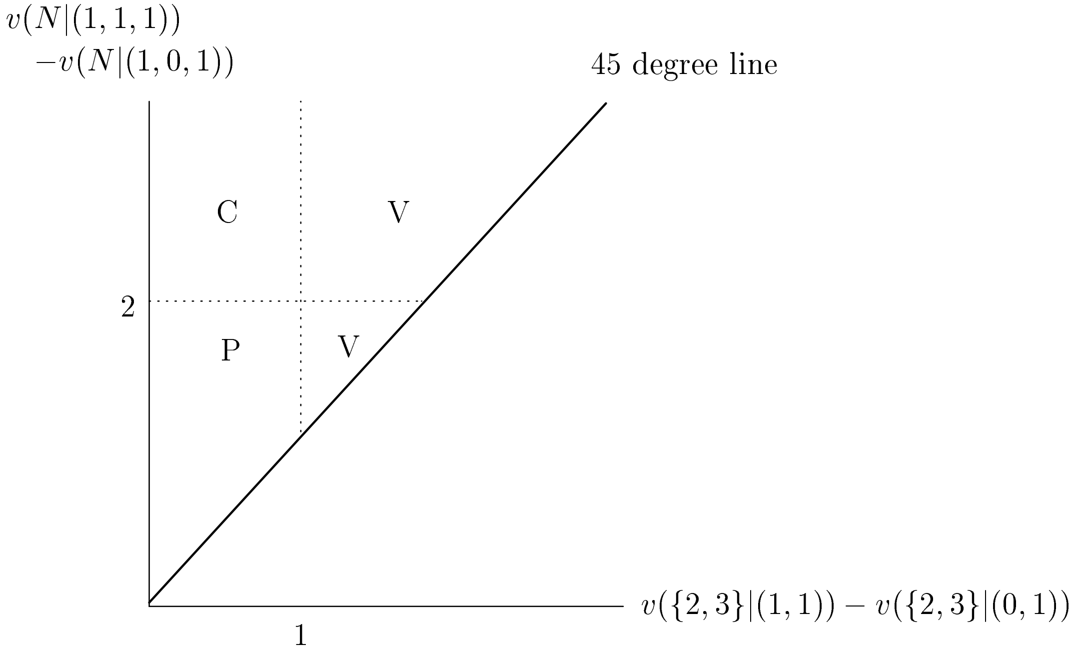


Figure 5 ($\alpha_2 < 0$, $\alpha_3 \geq 1$)



Region P: The pyramidal hierarchy is chosen.
Region V: The vertical hierarchy is chosen.
Region C: The common agency is chosen.