Noncooperative Jurisdiction Formation in a Local Public Goods Economy

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Abstract

In this paper, we develop a noncooperative bargaining model for forming jurisdictions and sharing the cost of a local public good. We derive a necessary and sufficient condition for the existence of a pure-strategy, Pareto-efficient stationary subgame perfect equilibrium when the discount factor is close to unity. In this equilibrium, individuals with similar tastes cluster together in their local jurisdiction. The relationship to Tiebout's hypothesis is examined.

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1 Introduction

In this paper, we develop a noncooperative bargaining model of endogenously formed jurisdictions in a local public good economy that has a finite number of individuals. There are many studies on local public good economies. In his seminal paper, Tiebout (1956) asserted that in a local public goods economy, individuals are forced to reveal their true preferences for the public goods through their choice of jurisdiction and that public goods are efficiently provided by each jurisdiction. Choosing jurisdictions to reveal their preferences for the public goods implies that individuals are '*voting with their feet.*' Moreover, he asserted that individuals are segregated into jurisdictions according to their preferences for local public goods through voting with their feet. Then, in equilibrium, each jurisdiction comprises individuals with similar tastes. In formal terms, Tiebout hypothesized that in a local public goods economy, there exists an equilibrium and it is Pareto efficient. Tiebout's paper stimulated much theoretical interest because his notion of equilibrium is not easily formalized. Bewley (1981) doubted the validity of Tiebout's assertion. He provided counter-examples to both existence and efficiency of equilibrium. It was widely recognized that an equilibrium might not exist because in a local public goods economy with a finite number of individuals, individuals who move from one jurisdiction to another have nonnegligible effects. This is known as the *integer problem* (see Wooders, 1978). To avoid the integer problem, many researchers assumed that there is a continuum of individuals or a large finite number of individuals in the economy (Ellickson *et al.*, 1999, 2001, Greenberg, 1983, Hammond *et al.*, 1989, and Wooders, 1980). In this paper, we analyze an economy with a finite number of individuals. Moreover, we assume that jurisdictions are endogenously formed.

Research on finite local public goods economies was initiated by Guesnerie and Oddou (1979, 1981), who assumed that jurisdictions are endogenously formed. They analyzed a finite local public goods economy in which the cost of the public good in each jurisdiction is financed by a proportional income tax. They introduced the notion of a *C-stable solution*. A C-stable solution is essentially a 'core'-like allocation. As is the core, a C-stable solution is immune to threats of secession by coalitions. Guesnerie and Oddou (1979) demonstrated the existence of a C-stable solution when there are up to three individuals. Guesnerie and Oddou (1981) derived sufficient conditions for the formation of a grand coalition. Greenberg and Weber (1986) showed that there is a C-stable solution for any number of individuals under restricted preferences domain. In their model, the ordering of all individuals can be described by a single parameter and individuals with similar tastes cluster together in equilibrium. Rather than adopting a cooperative game approach such as the core, Konishi *et al.* (1998) defined a normal-form game in which individuals choose levels of the public good. They showed that there exists a pure-strategy Nash equilibrium in a finite local public goods economy when there is a poll tax scheme. In addition, the Nash equilibrium does not satisfy a weak efficiency condition. In these studies of a finite local public goods economy, a tax scheme for financing the cost of the public good is restricted to a proportional income tax or a poll tax. Furthermore, congestion effects are ignored[1]. When there is a proportional income tax or a poll tax, individuals who move from one jurisdiction to another have nonnegligible effects on the tax base of both jurisdictions. These effects generate discontinuous jumps in the payoffs. Thus, fixed tax schemes may prevent equilibrium to exist. In this paper, we incorporate an arbitrary tax scheme. That is, the cost-sharing rule is endogenously determined by negotiations among members of each jurisdiction. Each jurisdiction can be considered as the group of individuals that can make binding agreements about the level of the local public good and the associated cost-sharing rule. Moreover, we consider congestion effects.

We examine a local public goods economy with a finite number of individuals. First, we present a noncooperative bargaining model for forming jurisdictions and sharing the cost of the local public good. Next, we provide a necessary and sufficient condition for the existence of a pure-strategy, Pareto-efficient stationary subgame-perfect equilibrium (SSPE). The condition can be understood with relation to the C-stable solution. It is assumed that the preferences domain is restricted to that represented by the quasi-linear utility function. Individuals can be ordered according to the strength of their preferences for the local public good. In addition, we incorporate '*anonymous crowding*,' under which each individual cares only about the number of agents in the same jurisdiction, but not about their characteristics. Given these assumptions, a Pareto-efficient coalition structure implies a partition of the set of individuals to coalitions that comprise individuals with similar tastes. A Pareto-efficient SSPE in our model would have the features described by Tiebout (1956). Focusing on the necessary and sufficient condition for the existence of a pure strategy, Pareto-efficient SSPE, we investigate the equilibrium coalition structure and examine how this relates to Tiebout's hypothesis.

Mutuswami *et al.* (2004) also used a noncooperative bargaining model to analyze a local public goods economy. They incorporated congestion effects and an arbitrary tax scheme. Hence, their framework is are similar to ours.

[1]Conley and Smith (2005) give a comprehensive survey of the many contributions in various forms of crowding or congestion effects.

However, our noncooperative extensive-form game is novel. Their bargaining game model is a '*bidding game*' in which individuals bid for the right to propose a coalition (jurisdiction) and a production plan for that coalition's public good. They propose the bidding game as a mechanism of generating efficient outcomes in the local public goods economy. By contrast, we use a noncooperative coalitional bargaining game model with random proposers, provided by Okada (1996). The model extends the Rubinstein's alternating-offer model to n-person coalitional bargaining. The key feature of the model is that a proposer is randomly selected with equal probability among active players in each round. This feature makes the model tractable and enables us to investigate the equilibrium coalition structure in economies with heterogeneous individuals. It is well known that even if individuals are homogeneous, the equilibrium coalition structure may be inefficient and complex in a fixed-order protocol bargaining model (see, for example, Chatterjee *et al.*, 1993, and Ray and Vohra, 1999).

We assume the same bargaining procedure as does Okada (1996). However, Okada only considered the bargaining situations described by superadditive games with transferable utility (TU and superadditive game). As shown later (Example 1), a game associating to a local public goods economy with congestion effects may not belong to the category of TU and superadditive games. Therefore, Okada's theorems cannot directly apply to a local public goods economy. We develop a new theorem for characterizing the situations in which the efficient coalition structure is attained. The generalized theorem relating to the existence of an SSPE in a game with a TU and *non-superadditive* game and related theorems are given in our companion paper, Miyakawa (2005).

The paper is organized as follows. In Section 2, we defines a local public good economy and introduces our noncooperative coalitional bargaining model. In Section 3, we characterizes the efficient coalition structure and derive a necessary and sufficient condition for the existence of a pure-strategy and efficient SSPE. In Section 3, we discuss the equilibrium coalition structure and its relationship to Tiebout's hypothesis. The proof of Theorem 2 is in the Appendix.

2 Local Public Good Economy

2.1 Basic framework

We consider an economy consisting of n individuals, the set of which is denoted by $N = \{1, \ldots, n\}$. There is one public good and one private good. We assume the constant returns to scale technology of the public good. In particular, the production technology permits the transformation of one unit of private good into one unit of public good. Each individual $i \in N$ is endowed with a positive amount I_i of the private good. Here, initial endowments of the private good is identical for all individuals[2], i.e., $I_i = I$ for all $i \in N$. Individual i in coalition S has preferences, represented by the quasi-linear utility function with congestion effects:

$$u_i(g) - c(|S|) + x_i, \tag{1}$$

where $u_i(g)$ is utility from the local public good g, and x_i is her private good consumption. The term $-c(|S|)$ is disutility from congestion, where $|S|$ denotes the cardinality of the coalition S. The function u_i and c are assumed to be continuous and increasing. Note that the level of congestion, $c(|S|)$, depends only on the number of individuals in coalition S and is common to all members of S. Models of a local public goods economy with this type of congestion, which is called '*anonymous crowding*', are provided by Scotchmer and Wooders (1987), and Konishi, Le Breton and Weber (1997). We assume that the utility of the public good is separable from congestion effects and that the disutility from congestion is common among the members of the coalition. These assumptions are very restricted ones. Due to these assumptions, however, all individuals can be ordered according to the strength of their preferences for the public good. Formally, we assume the utility function of the local public good for each individual to satisfy the following assumption;

Assumption 1 (Preserving the Order of Players). If $i < j$ for any two player $i, j \in N = \{1, 2, \ldots, n\}$, then the inequality $u_i(g) \leq u_j(g)$ holds for all $g \in \mathbb{R}_+$. Moreover, the difference $u_j(g) - u_i(g)$ is strictly increasing with g if $u_j(g) \neq u_i(g)$.

The public good is produced by each coalition. A coalition can be regarded as a local jurisdiction. The cost of public good can be financed through an arbitrary tax on the members of coalition. The coalition alone cannot benefit from the public good produced by other coalitions. Thus, there is no spillover effect between the local jurisdictions. In previous studies in a local public goods economy, cost sharing rules (tax schemes) were restricted to a proportional income tax or a poll tax (see Guesnerie and Oddou, 1979 and 1981, Greenberg and Weber, 1982 and 1986, Konishi, Le Breton

[2]Because we assume that each individual has a quasi-linear preferences, the assumption of identical endownments is not crusial for our results. Even if individuals have different incomes, the same results are obtained.

and Weber, 1998). We allow an arbitrary tax scheme here. The tax burdens to each individual in the coalition are determined by negotiations among the members of the coalition. We will construct a noncooperative bargaining game model to determine the coalition and the cost-sharing of the public good, and investigate a Nash equilibrium point satisfying some properties. The coalitional form game (N, v) to describe the bargaining situation is convenient to derive the equilibrium of the noncooperative game.

The coalitional form game (N, v) associating to the local public good economy is defined by; for each $S \subset N$,

$$v(S) = \max_{g \in \mathbb{R}} \left\{ \sum_{i \in S} u_i(g) - |S|c(|S|) + |S|I - g \right\}. \tag{2}$$

The value $v(S)$ denotes the maximum total payoff of the members of coalition S by producing the local public good. Since each individual has a quasi-linear utility function, one unit of utility for a consumer can be tranferred to another consumer through reallocation of the private good. Therefore, the game (N, v) is a game in coalitional form with transferable utility (TU game).

Definition 1. A TU game (N, v) is called *superadditive* if, for all $S, T \subset N$, $S \cap T = \emptyset$,

$$v(S) + v(T) \leq v(S \cup T).$$

Definition 2. The grand coalition N is called *universally efficient* if

$$v(N) \geq \sum_{k=1}^{K} v(S_k), \text{ for every partition } \{S_1, \ldots, S_K\} \text{ of } N.$$

It is easy to show that if the grand coalition is universally efficient then it is superadditive. In our game defined by (2), it is not necessarily superadditive and N is not universally efficient.

Example 1. The economy consists of four individuals $N = \{1, 2, 3, 4\}$. Every individual has an income of 5, i.e., $I = 5$. Utility functions of the public good are: $u_1(g) = u_2(g) = 4 \log g$, $u_3(g) = u_4(g) = 2 \log g$, where log represents the natural logarithm. Congestion costs depending on the number of individuals are given by: $c(1) = 1$, $c(2) = 3$, $c(3) = 10$, and $c(4) = 15$. By definition of (2), the characteristic function for coalition $\{1\}$ and $\{2\}$ is

$$v(\{1\}) = v(\{2\}) = \max_{g} \left(4 \log g - c(1) + I - g \right).$$

Because the maximum is attained at $g = 4$, it is reduced to

$$v(\{1\}) = v(\{2\}) = 4\log 4 - 1 + 5 - 4 = 8\log 2.$$

Similarly, $v(\{3\}) = v(\{4\}) = \max_g(2\log g - c(1) + I - g)$. We can obtain the maximum at $g = 2$. Then

$$v(\{3\}) = v(\{4\}) = 2\log 2 - 1 + 5 - 2 = 2\log 2 + 2.$$

By carrying out the same procedures, we can derive other characteristic functions as follows:

$$\begin{aligned}
v(\{1,2\}) &= \max_g (4\log g + 4\log g - c(2) + 2I - g) \\
&= 4\log 8 + 4\log 8 - 3 + 10 - 8 = 24\log 2 - 1, \\
v(\{3,4\}) &= \max_g (2\log g + 2\log g - c(2) + 2I - g) \\
&= 2\log 4 + 2\log 4 - 3 + 10 - 4 = 8\log 2 + 3, \\
v(\{1,2,3\}) &= v(\{1,2,4\}) = \max_g (8\log g + 2\log g - c(3) + 3I - g) \\
&= 8\log 10 + 2\log 10 - 10 + 15 - 10 = 10\log 10 - 5, \\
v(\{1,3,4\}) &= v(\{2,3,4\}) = \max_g (4\log g + 4\log g - c(3) + 3I - g) \\
&= 8\log 8 - 10 + 15 - 8 = 24\log 2 - 3, \\
v(\{1,2,3,4\}) &= \max_g (8\log g + 4\log g - c(4) + 4I - g) \\
&= 12\log 12 - 15 + 20 - 12 = 12\log 12 - 7.
\end{aligned}$$

Since $\log 2 = 0.6931\cdots$, $\log 10 = 2.3025\cdots$, and $\log 12 = 2.4848\cdots$, then the values of the characteristic function are $v(\{1\}) = v(\{2\}) = 5.5448\cdots$, $v(\{3\}) = v(\{4\}) = 3.3862\cdots$, $v(\{1,2\}) = 15.6344\cdots$, $v(\{3,4\}) = 8.5448\cdots$, $v(\{1,2,3\}) = v(\{1,2,4\}) = 18.025\cdots$, $v(\{1,3,4\}) = v(\{2,3,4\}) = 13.6344\cdots$, and $v(\{1,2,3,4\}) = 22.8176\cdots$. As a result, we obtain

$$\begin{aligned}
&v(\{1,2\}) + v(\{3\}) = 19.0206\cdots > v(\{1,2,3\}) = 18.025\cdots, \\
&v(\{1\}) + v(\{3,4\}) = 14.0896\cdots > v(\{1,2,3\}) = 13.6344\cdots, \\
&v(\{1,2\}) + v(\{3,4\}) = 24.1792\cdots > v(\{1,2,3,4\}) = 22.8176\cdots.
\end{aligned}$$

Thus, the above game is not superadditive and N is not universally efficient.

If there is no congestion effect, the characteristic function is given by

$$v(S) = \max_{g \in G} \left\{ \sum_{i \in S} u_i(g) - |S|I - g \right\}. \qquad (3)$$

In this case the game is superadditive and the grand coalition is universally efficient.

2.2 Noncooperative coalitional bargaining

We now define a noncooperative bargaining game model for constituting local jurisdictions. We have to adopt the bargaining procedure based on the extensive form game in order to determine both the coalition structure and the tax burden of each jurisdiction endogenously. In the bargaining game, the individuals are partitioned into local jurisdictions, and each jurisdiction produces his own public good and shares its production costs among the members of the coalition. Since all cost sharing rules are allowed (a tax scheme is not restricted at all), it is possible for the members of the local jurisdiction to select any feasible payoff distribution in the coalition. In addition, every proposer inevitably chooses the level of public good as to maximize the sum of payoffs (money) for the members of the coalition S. Thus, the problem of determining a coalition, the level of local public good and its cost-sharing is reduced to the problem of choosing a feasible payoff allocation $y^S \in v(S)$.

A payoff vector for a coalition S is denoted by $y^S = (y_i^S)_{i \in S} \in \mathbb{R}^{|S|}$. A payoff vector y^S for S is called *feasible* if

$$\sum_{i \in S} y_i^S \leq v(S).$$

We denote by Y^S the set of all feasible payoff vectors for S.

Our noncooperative bargaining model proceeds as follows.

At every round $t = 1, 2, \ldots$, one player is selected as a proposer with equal probability among all player still active in bargaining. Let N^t be the set of all active players at round t, where the bargaining starts with all players at round 1, i.e., $N^1 = N$.

The proposer i chooses a coalition S with $i \in S \subset N^t$ and a payoff vector $y^S \in Y^S$. All other players in S accept or reject the proposal sequentially. If all the other players in the coalition accept the proposal, then it is agreed upon and enforced. This implies the emergence of a local jurisdiction which consists of the members of coalition S and has a local public good and a cost sharing scheme that yield the payoff vector y^S. The remaining players outside S can continue negotiations at the next round. Thus, $N^{t+1} = N^t \backslash S$. If some players in S reject the proposal, then negotiations go on to the next round and a new proposer randomly selected by the same rule. In this case, $N^{t+1} = N^t$. The bargaining continues until there exists a subset S of active players such that $v(S) > 0$. Consequently, the individuals in N are partitioned into local jurisdictions. We can regard a coalition as a local jurisdiction and a coalition structure of N as a jurisdiction structure.

When a proposal (S, y^S) is agreed upon at round t, the payoff of every member $i \in S$ is $\delta^{t-1} y_i^S$, where δ is a discount factor, and $0 \leq \delta < 1$. For players who do not belong to any coalitions, their payoffs are assumed to be zero. Every player has perfect information.

Our bargaining procedure is same as in Okada (1996). Because a proposer is randomly selected, his model is called "random proposer" model. He, however, considered only a bargaining situation described by a superadditive coalitional game. The game to a local public good economy is not necessarily superadditive as shown in Example 1.

We denote by $\Gamma^S(\delta)$ the bargaining model with the player set S. Γ^S is used when the discount factor δ converges to one; $\Gamma^S(\delta) \to \Gamma^S$ as $\delta \to 1$. Let σ_i be a strategy for player i in $\Gamma^N(\delta)$ and $\sigma = (\sigma_1, \ldots, \sigma_n)$ be a strategy combination.

We shall adopt the following solution concept.

Definition 3. (i) A strategy combination σ^* of the game $\Gamma^N(\delta)$ is called a *stationary subgame perfect equilibrium point* (SSPE) if it is a subgame perfect equilibrium point with the property that for every $t = 1, 2, \ldots,$ the tth round strategy of every player depends only on the set N^t of all players active at round t. (ii) A strategy combination σ^* of the game Γ^N is called a *limit* SSPE if it is a limit point of SSPEs of $\Gamma^N(\delta)$ as $\delta \to 1$.

For an SSPE σ of $\Gamma^N(\delta)$ and every coalition $S \subset N$, let $v^S = (v_i^S)_{i \in S}$ denotes the expected payoff vector of players for σ in the subgame $\Gamma^S(\delta)$, and $\theta^S = (T_i^S)_{i \in S}$ be the collection of coalitions T_i^S proposed by every player i on the plays of σ in $\Gamma^S(\delta)$. We call the collection $\{(v^S, \theta^S) \mid S \subset N\}$ the *configuration* of the SSPE σ.

3 Main Results

3.1 Efficiency and Existence

Let characterize the efficient coalition structure in our model. We denote the set of all partitions of S by

$$\Pi(S) = \left\{ \{S_1, \ldots, S_K\} \;\middle|\; \bigcup_{k=1}^{K} S_k = S, \text{ and } S_i \cap S_j = \emptyset, i \neq j \right\}.$$

A element $\pi^S = \{S_1, \ldots, S_K\} \in \Pi(S)$ is called a *coalition structure* of S. For a given coalition structure $\pi^S = \{S_1, \ldots, S_K\}$, we can define the function

$$V(\pi^S; S) = \sum_{k=1}^{K} v(S_k).$$

Then, the notion of efficient coalition structure is defined as follows.

Definition 4. A coalition structure π is called an *efficient coalition structure* of S if, for all $\pi' \in \Pi(S)$, $V(\pi; S) \geq V(\pi'; S)$.

An efficient coalition structure of S always exist in a game with transferable utility. But multiple coalition structures may be efficient. It is easy to see that an efficient coalition structure of N is $\{N\}$ in the case where the grand coalition is universally efficient.

Next let us define the notion of connectedness about a coalition. Note that player i and player j are ordered by $i < j$ only if $u_i(g) < u_j(g)$ or $u_i(g) = u_j(g)$ for all g.

Definition 5. A coalition S is called *connected* if, for any three players i, j, k with $i < j < k$ and $u_i(g) \neq u_k(g)$, $i, k \in S$ imply $j \in S$. Moreover, the *connected coalition structure* of S is the partition of S to connected coalitions.

In the connected coalition structure, each jurisdiction (coalition) is an interval of players, i.e., individuals with similar tastes for the public good cluster together.

We can prove that an efficient coalition structure is connected in our setting.

Theorem 1. *An efficient coalition structure of S is connected in the local public good economy.*

Proof. Let $\pi^*(S) = \{T_1(S), \ldots, T_K(S)\}$ be an efficient coalition structure of S. Suppose that the efficient coalition structure is not connected. By definition, there exists $T_k(S) \in \pi^*(S)$ such that, for some $i, j \in T_k(S)$, $i < h < j$ and $h \notin T_k(S)$. Let $T_\ell(S)$ be the coalition that includes individual h and let $g_k^*(S)$ and $g_\ell^*(S)$ be the public good levels which achieve

$$v(T_k(S)) = \max_g \left(\sum_{m \in T_k(S)} u_m(g) - |T_k(S)| c(|T_k(S)|) - g \right), \text{ and}$$

$$v(T_\ell(S)) = \max_g \left(\sum_{m \in T_\ell(S)} u_m(g) - |T_\ell(S)| c(|T_\ell(S)|) - g \right),$$

respectively.

Consider the case of $g_k^*(S) \leq g_\ell^*(S)$. Let replace individual j from coalition $T_k(S)$ to $T_\ell(S)$ and do individual h from $T_\ell(S)$ to $T_k(S)$. Thus, $T_k(S)$ ($T_\ell(S)$) changes to $(T_k(S)\setminus\{h\})\cup\{j\}$ ($(T_\ell(S)\setminus\{j\})\cup\{h\}$). If $g_k^*(S)$ and $g_\ell^*(S)$ are fixed, then the value of

$$\sum_{m\in(T_\ell(S)\setminus\{h\})\cup\{j\}} u_m(g_\ell^*(S)) - |(T_\ell(S)\setminus\{h\})\cup\{j\}|c(|(T_\ell(S)\setminus\{h\})\cup\{j\}|) - g_\ell^*(S)$$

is larger than that of $v(T_\ell(S))$ by $(u_j(g_\ell^*(S)) - u_h(g_\ell^*(S)))$. In addition, the value of

$$\sum_{m\in(T_k(S)\setminus\{j\})\cup\{h\}} u_m(g_k^*(S)) - |(T_k(S)\setminus\{j\})\cup\{h\}|c(|(T_k(S)\setminus\{j\})\cup\{k\}|) - g_k^*(S)$$

is smaller than $v(T_k(S))$ by $(u_j(g_k^*(S)) - u_h(g_k^*(S)))$.

By Assumption 1, $(u_j(g_\ell^*(S)) - u_h(g_\ell^*(S)))$ is larger than $(u_j(g_k^*(S)) - u_h(g_k^*(S)))$. This contradicts the assumption that $\pi^*(S)$ is an efficient coalition structure.

Next consider the case of $g_k^*(S) > g_\ell^*(S)$. In this case let us transpose individual i from $T_k(S)$ to $T_\ell(S)$ and individual h from $T_\ell(S)$ to $T_k(S)$. Thus, we permute between i and h in a coalition structure $\pi^*(S)$. Then, even if the public good level remains at $g_k^*(S)$, the value of

$$\sum_{m\in(T_k(S)\setminus\{i\})\cup\{h\}} u_m(g_k^*(S)) - |(T_k(S)\setminus\{i\})\cup\{h\}|c(|(T_k(S)\setminus\{i\})\cup\{k\}|) - g_k^*(S)$$

is larger than $v(T_k(S))$ by $(u_k(g_k^*(S)) - u_i(g_k^*(S)))$. On the other hand, if $g_\ell^*(S)$ is fixed, the value of

$$\sum_{m\in(T_\ell(S)\setminus\{h\})\cup\{i\}} u_m(g_\ell^*(S)) - |(T_\ell(S)\setminus\{h\})\cup\{i\}|c(|(T_\ell(S)\setminus\{h\})\cup\{i\}|) - g_\ell^*(S)$$

is smaller than $v(T_\ell(S))$ by $(u_h(g_\ell^*(S)) - u_i(g_\ell^*(S)))$. The former exceeds the latter by Assumption 1. Thus, coalition $(T_k(S)\setminus\{i\})\cup\{h\}$ and $(T_\ell(S)\setminus\{h\})\cup\{i\}$ realize the larger worth in aggregate than $v(T_k(S)) + v(T_\ell(S))$. This is a contradiction. □

Theorem 1 says that a Pareto efficient allocation leads to the coalition structure in which individuals with similar taste for the public good is clustering together. We are now ready to investigate an SSPE in which an efficient allocation is realized.

Definition 6. An SSPE σ of the game $\Gamma^N(\delta)$ is called *subgame coalitional efficient* if, for every subgame $\Gamma^S(\delta)$, every player $i \in S$ proposes the coalition which is a component of the efficient coalition structure of S in σ. A *limit subgame coalitional efficient* SSPE of Γ^N is defined to be a limit point of subgame coalitional efficient SSPEs of $\Gamma^N(\delta)$ as $\delta \to 1$.

The notion of subgame coalitional efficiency is a generalization of *subgame efficiency* in Okada (1996). When a game is superadditive, every player proposes the full coalition S in a subgame coalitional efficient SSPE σ for every subgame $\Gamma^S(\delta)$. This implies the subgame efficiency. Note that the notion of subgame coalitional efficiency is stronger than the Pareto efficiency of the expected payoff vector for n players in $\Gamma^N(\delta)$. It requires that in all subgames $\Gamma^S(\delta)$, $S \subset N$, the efficient coalition structures must be established.

Tiebout (1956) argues that, if there were enough jurisdictions, individuals would choose their most preferable jurisdictions as a place in which to live (voting with their feet), and such a voting generates the Pareto efficient allocation. Furthermore, he asserts, because individuals reveal their preference for local public goods by choice of jurisdiction, all those deciding to live in the same jurisdiction would have the similar tastes. Theorem 1 shows that in our local public good economy an efficient coalition structure implies clustering of individuals with similar tastes. By definition, the efficient coalition structure can be realized in a subgame coalitional efficient SSPE. We investigate the situation where a discount factor δ is sufficiently close to 1. In this case, the ex-ante expected equilibrium payoff vector converges to the ex-post expected equilibrium payoff vector because all individuals propose the same proposal.

In addition, we would like to guarantee the existence of a pure-strategy Nash equilibrium, rather than a mixed-strategy equilibrium. If mixed strategies about the choice of a coalition is allowed, the existence of an SSPE can be proved in a noncooperative bargaining game model with the general non-superadditive TU game, see our companion paper, Miyakawa (2005). Non-existence of equilibrium in a local public good economy comes from the discrete choice of a jurisdiction to live, which is called 'integer problem'. Allowing mixed strategies about the choice of a jurisdiction is not a direct solution of the integer problem. Therefore, we specify the necessary and sufficient condition for the existence of a pure-strategy SSPE with subgame coalitional efficiency. A necessary and sufficient condition for the existence of a pure-strategy and subgame efficient SSPE has been provided by Okada (1996). But he only considered the bargaining situation described by a TU and superadditive game. The local public good economy may become a TU but non-superadditive game. Then, Okada's condition does not directly apply to our bargaining situation.

For each $S \subset N$, we represent the efficient coalition structure of S by $\pi^*(S) = \{S_1^*(S), \ldots, S_{K^S}^*(S)\}$. In a TU and non-superadditive game, the following theorem is established :

Theorem 2. *There exists a pure-strategy and limit subgame coalitional efficient SSPE of* Γ^N *if and only if the game* (N, v) *satisfies; for all* $S \subset N$,

$$\frac{v(S_1^*(S))}{|S_1^*(S)|} \geq \max_{T \subset N, i \in T} \left(v(T) - \sum_{j \in T, j \neq i} y_j \right) \text{sub.to } y_j = \frac{v(S_k^*(S))}{|S_k^*(S)|}, \tag{4}$$

$$j \in S_k^*(S) \cap T, k = 1, 2, \ldots, K^S, \text{ for all } i \in S_1^*(S),$$

$$\vdots$$

$$\frac{v(S_{K^S}^*(S))}{|S_{K^S}^*(S)|} \geq \max_{T \subset N, i \in T} \left(v(T) - \sum_{j \in T, j \neq i} y_j \right) \text{sub.to } y_j = \frac{v(S_k^*(S))}{|S_k^*(S)|},$$

$$j \in S_k^*(S) \cap T, k = 1, 2, \ldots, K^S, \text{ for all } i \in S_{K^S}^*(S).$$

The expected equilibrium payoff vector $(v_j^*)_{j \in S}$ *in* Γ^S *is given by*

$$v_i^* = \frac{v(S_1^*(S))}{|S_1^*(S)|}, \ \forall\ i \in S_1^*(S), \ldots, v_i^* = \frac{v(S_{K^S}^*(S))}{|S_{K^S}^*(S)|}, \ \forall\ i \in S_{K^S}^*(S). \tag{5}$$

Proof: See our companion paper, Miyakawa (2005). We also give a proof in Appendix in order to make this paper self-contained . □

The local public good economy belongs to a class of TU and non-superadditive game. So, we can applies Theorem 2 to the local public good economy.

Theorem 2 shows that there exists a pure strategy and limit subgame coalitional efficient SSPE if and only if each individual obtains the maximum payoff by forming coalition $S_k^*(S)$ under the condition that other individual j must be guaranteed to get his payoff $v(S_\ell^*(S))/|S_\ell^*(S)|$, where $S_\ell^*(S) \in \pi^*(S)$ and $j \in S_\ell^*(S)$, if j is the member of coalition. A pure-strategy and limit subgame coalitional efficient SSPE in the local public good economy realizes a Pareto efficient allocation and a coalition structure such that individuals with similar taste cluster together. Note that, in an SSPE, individual $i \in S_k^*$, $k = 1, 2, \ldots, K^N$, shares the worth $v(S_k^*)$ equally among the members of the coalition S_k^*, where $S_k^* \in \pi^*(N)$.

Moreover, the condition says that there exists no group of individuals who can benefit by deviating from the efficient coalition structure $\pi^*(S)$. This means that the payoff vector (5) is in the *C-stable solution* defined by Guesnerie and Oddou (1979, 1981). Note that a C-stable solution here is not coincident with a C-stable solution in Guesnerie and Oddou's model because they restrict a tax system to a proportional income tax or a equal share tax.

3.2 Coalition structure

We examine the equilibrium coalition structure. First let give a simple example in order to clarify the situation that satisfies the condition (4) in Theorem 2.

Example 2. Consider a set of individuals $N = \{1, 2, 3\}$ and an income level $I = 8$ for all individuals. The congestion costs are assumed to be $c(1) = 0$, $c(2) = 2$, and $c(3) = 10$.

(i) If $u_1(g) = u_2(g) = 6 \log g$ and $u_3(g) = 2 \log g$, then, the values of characteristic function are given by

$$
\begin{aligned}
&v(\{1\}) = v(\{2\}) = 6 \log 6 + 8 - 6 = 12.7502 \cdots, \\
&v(\{3\}) = 2 \log 2 + 8 - 2 = 7.3862 \cdots, \\
&v(\{1, 2\}) = 12 \log 12 - 2 + 16 - 12 = 31.8176 \cdots, \\
&v(\{1, 3\}) = v(\{2, 3\}) = 8 \log 8 - 2 + 16 - 8 = 24.6944 \cdots, \\
&v(\{1, 2, 3\}) = 14 \log 14 - 10 + 24 - 14 = 36.946 \cdots.
\end{aligned}
$$

In this case, the efficient coalition structure is $\{\{1, 2\}, \{3\}\}$. Therefore, we can calculate the equilibrium payoff vector $v_1^* = v(\{1, 2\})/2 = 15.9088 \cdots$, $v_2^* = v(\{1, 2\})/2 = 15.9088 \cdots$, $v_3^* = v(\{3\}) = 7.3862 \cdots$. It is sufficient to check that the above payoff for each individual is greater than the payoff obtained by proposing other coalition while other individuals are conditional to obtain the continuation payoffs. Let us consider $i = 1$. When the proposed coalition is $T = \{1, 2, 3\}$, individual $i = 1$ has

$$v(\{1, 2, 3\}) - v_2^* - v_3^* = 13.651 \cdots.$$

When $T = \{1, 3\}$, individual $i = 1$ can obtains

$$v(\{1, 3\}) - v_3^* = 15.3082 \cdots.$$

If $T = \{1\}$, individual $i = 1$ has $12.7502 \cdots$. All values are smaller than $v_1^* = 15.9088$. Thus, the condition 4 in Theorem 2 for individual $i = 1$ is satisfied. The same calculation is applied to individual $i = 2$. So, the condition for individual $i = 2$ is also satisfied. Next consider $i = 3$. If individual $i = 3$ proposes the coalition $T = \{1, 2, 3\}$, he obtains

$$v(\{1, 2, 3\}) - v_1^* - v_2^* = 6.3196 \cdots.$$

If either $T = \{1, 3\}$ or $T = \{2, 3\}$, then, $i = 3$ has

$$v(\{1, 3\}) - v_1^* = v(\{2, 3\}) - v_2^* = 7.3812 \ldots.$$

These payoffs are smaller than $v_3^* = 7.3862$. The condition is also satisfied for $i = 3$.

(ii) Next, assume that $u_1(g) = u_2(g) = 6 \log g$, and $u_3(g) = 3 \log g$. The diversity of the preferences for the public good is narrower than case (i) because individual 3 is changed from $2 \log g$ to $3 \log g$ and individual $i = 1$ and $i = 2$ are unchanged. The efficient coalition structure is $\{\{1,2\},\{3\}\}$. The payoffs to check the conditions are $v_1^* = v_2^* = v(\{1,2\})/2 = 15.9088\cdots$ and $v_3^* = v(\{3\}) = 8.2958\cdots$. Focus on individual $i = 1$. When $T = \{1,3\}$, individual $i = 1$ has

$$v(\{1,3\}) - v_3^* = 16.479\cdots.$$

This is greater than $v_1^* = 15.9088\cdots$. The condition for individual $i = 1$ is violated.

(iii) Finally, assume that $u_1(g) = 7 \log g$, $u_2(g) = 6 \log g$, and $u_3(g) = 2 \log g$. In this case, the diversity of the preferences for the public good is wider than case (i). The efficient coalition structure is also $\{\{1,2\},\{3\}\}$. The targeted payoff vector is $v_1^* = v_2^* = v(\{1,2\})/2 = 16.1712$ and $v_3^* = v(\{3\}) = 7.3862\cdots$. Check the condition for individual $i = 1$. If individual $i = 1$ proposes $T = \{1,3\}$, then he can obtains

$$v(\{1,3\}) - v_3^* = 17.3886\cdots.$$

The condition (4) is not satisfied.

Example 2 shows that the condition (4) in Theorem 2 is very sensitive and is difficult to be satisfied.

Next let us consider the case without congestion effects. In this case the game becomes superadditive. If a game is superadditive, Theorem 2 is reduced to the next corollary. This corollary is same as Theorem 3 in Okada (1996).

Corollary 1 (Okada (1996)). *There exists a limit subgame efficient (pure strategy) SSPE of Γ^N uniquely if and only if the game satisfies*

$$\frac{v(S)}{|S|} \geq \frac{v(T)}{|T|} \quad \textit{for all coalitions } S \textit{ and } T \textit{ of } N \textit{ with } T \subset S. \tag{6}$$

The expected equilibrium payoff vector $v^S = (v_i^S)_{i \in S}$ in every subgame $\Gamma^S(\delta)$ is given by $(v(S)/|S|, \ldots, v(S)/|S|)$.

According to Corollary 1, the full coalition may not be formed even if there is no congestion and no restriction on a tax scheme (that is, the game is supperadditive and the grand coalition is universally efficient). Strategic coalition formation is a source of inefficiency in the local public goods economy. Let us give an example where the condition in Corollary 1 does not be satisfied.

Example 3. Consider $N = \{1, 2, 3\}$ and $u_1(g) = u_2(g) = 8 \log g$, $u_3(g) = 2 \log g$. Every individual has an income of 10, i.e., $I = 10$, and there is no congestion cost; $c(1) = c(2) = c(3) = 0$. We can easily derive the following characteristic functions:

$$\begin{aligned}
&v(\{1\}) = v(\{2\}) = 24 \log 2 + 2 = 18.6334 \cdots , \\
&v(\{3\}) = 2 \log 2 + 8 = 9.3862 \cdots , \\
&v(\{1, 2\}) = 64 \log 2 + 4 = 48.3583 \cdots , \\
&v(\{1, 2, 3\}) = 18(\log 2 + 2 \log 3) + 12 = 64.0254 \cdots .
\end{aligned}$$

Then $v(\{1, 2, 3\})/3 < v(\{1, 2\})/2$. Although the game is superadditive, it does not satisfy the condition in Corollary 1.

If all individuals are identical and there is no congestion, the condition (6) is satisfied. Thus, the grand coalition is formed in the equilibrium. This result is in contrast with one provided by Chatterjee *et al.* (1993) and Ray and Vohra (1999). In their model the grand coalition is not necessary formed even if all individuals are identical, which is caused by the fixed proposal order over the players. On the other hand, the grand coalition is formed in the random-proposer model.

3.3 Discussion about Tiebout's Hypothesis

In this paper, we present a noncooperative bargaining model for forming jurisdictions and for sharing the cost of the local public good. We focus on a pure strategy, limit subgame coalitional efficient SSPE in the bargaining game. This SSPE generates the Pareto efficient allocation and leads to a cluster of individuals with similar tastes of the public good. Tiebout (1956) asserted that in a local public goods economy, there exists an equilibrium and it is Pareto efficient. Moreover, he argued that the true preferences of individuals for the public goods is revealed through the mechanism of the 'voting with their feet.' We explain how our results relate to Tiebout's assertions.

(i) Does Theorem 2 support the Tiebout's assertion? The answer is negative. Theorem 2 clarifies the necessary and sufficient condition for the existence of a pure-strategy, limit subgame coalitional efficient SSPE in a local public good economy with a finite number of individuals. Although a pure-strategy, Pareto-efficient SSPE in our model has the features described by Tiebout, the necessary and sufficient condition for its existence is unlikely to be satisfied. Example 2 indicates that the condition (4) is only satisfied in exceptional cases. Therefore, Theorem 2 shows that the equilibrium outcomes described by Tiebout are unlikely to emerge even when the formation of the jurisdiction and cost-sharing rules for the public good are endogenenously determined through negotiations.

(ii) Tiebout emphasized that preferences for the public good are revealed through the choice of the jurisdictions. A key feature of Tiebout's model is that inhabitants have free mobility. However, we treat a local jurisdiction as the group of individuals that can make binding agreements about the level of the public good and its associated cost-sharing. In particular, cost sharing is determined by unanimity among members of the jurisdiction. Personal tax burdens are determined by cotracts between individuals and the jurisdiction. Hence, the aspects of free mobility disappear in our model. If the cost-sharing rule takes a form of a proportional income tax or an equal share tax, the notion of free mobility can be introduced even in the coalitional bargaining model. This issue is for the future research.

(iii) As already mentioned, the notion of subgame coalitional efficiency is stronger than the Pareto efficiency. Tiebout only required the Pareto efficiency for an equilibrium outcome. If the condition (4) is satisfied, there is a subgame coalitional efficient SSPE. Therefore, the condition (4) might be unnecessarily strong. We have a room for weaken the condition to achieve the Pareto efficient allocation in equilibrium.

Appendix

Proof of Theorem 2. We provide two lemmas before giving the proof of Theorem 2. In addition, we focus on a class of payoff configuration in these lemmas.

Definition 7. A payoff configuration $\{v^S \mid S \subset N\}$ is called *feasible in the efficient coalition structure* $\pi^*(S) = \{S_1^*(S), \ldots, S_{K^S}^*\}$ if, for every S,

$$\sum_{j \in S_\ell^*(S)} v_j^S \leq v(S_\ell^*(S)), \quad \ell = 1, \ldots, K^S.$$

The first lemma shows that, in every pure strategy SSPE those payoff configuration is feasible in the efficient coalition structure, an agreement is made in the first round. We have to restrict a class of SSPE to prove no delay of agreement in equilibrium. If a game is superadditive, this restriction is unnecessary; no delay occurs in equilibrium, see Okada (1996).

Lemma 1. *In every pure strategy SSPE σ of $\Gamma^N(\delta)$ with $\{(v^S, \theta^S) \mid S \subset N\}$ such that the payoff configuration is feasible in the efficient coalition structure, every player $i \in N$ proposes at round 1 a solution (S_i, y^{S_i}) of the maximization problem:*

$$\max_{S, y^i} \left(v(S) - \sum_{j \in S, j \neq i} y_j^i \right) \tag{7}$$

$$\text{subject to } y_j^i \geq \delta v_j^N, \ \textit{for all } j \in S, j \neq i,$$
$$S \in \mathcal{S}_i.$$

Moreover, the proposal (S_i, y^{S_i}) is accepted in σ.

Proof. Let $x^i = (x_1^i, \ldots, x_n^i)$ be the expected equilibrium payoff vector when player i becomes the proposer at round 1. By definition of $\Gamma^N(\delta)$, $v_i^N = \sum_{k \in N} x_i^k / n$ for all $i \in N$. We denote by m^i the maximum value of (7).

We will prove that $x_i^i = m^i$.

($x_i^i \leq m^i$): Suppose that player i proposes (S, y^S) at round 1 such that $y_i^S > m^i$. Since m^i is the maximum value of (7), for some $j \in S$ with $j \neq i$, $y_j^S < \delta v_j^N$. Let j^* be the last responder of such a kind. In equilibrium the following is possible: (i) some responder after j^* reject i's proposal, and (ii) otherwise. If player j accepts the proposal in the case of (ii), then the proposal is agreed upon and player j obtain $y_{j^*}^S$ less than his continuation payoff δv_j^N. Therefore, it is optimal for j^* to reject i's proposal. Thus, i's proposal is rejected and the game goes on to round 2 whichever case occurs. Then, player i obtains the discount payoff δv_i^N.

The efficient coalition structure of N is denoted by $\pi^*(N) = \{S_1^*(N), \ldots, S_{K^N}^*\}$. Because we focus on a SSPE those payoff configuration is feasible in the efficient coalition structure, we have

$$\sum_{j \in S_\ell^*(N)} v_j^N \leq v(S_\ell^*(N)), \text{ for } \ell = 1, \ldots, K^N.$$

Thus, the pair $(S_\ell^*(N), (v_j^N)_{j \in S_\ell^*(N)})$, where $i \in S_\ell^*(N)$, satisfies constraints of the problem (7). This implies that $v_i^N \leq m^i$.

Since $V(\{i\}) \geq 0$ for all $i \in N$, every player i surely obtain more than zero as a payoff when i becomes the proposer. Therefore, $x_i^i \geq 0$. The responder also rejects the proposal in which his payoff is less than zero. Thus, $x_k^i \geq 0$. We have $v_i^N \geq 0$ because the above argument applies to all $i \in N$. Hence, $\delta v_i^N \leq v_i^N \leq m^i$. Player i obtains only δv_i^N even if he proposes a payoff greater than m^i. This implies $x_i^i \leq m^i$.

($x_i^i \geq m^i$): By the assumption that the payoff configuration is feasible in the efficient coalition structure, the pair $(S_\ell^*(N), (v_j^N)_{j \in S_\ell^*(N)})$ is a feasible solution of the problem (7). Then, the pair $(S_\ell^*(N), (\delta v_j^N)_{j \in S_\ell^*(N)})$ is also a feasible solution of the problem (7) because $0 \leq \delta \leq 1$. Therefore, $m^i \geq v_i^N \geq \delta v_i^N$. Suppose that $m^i = 0$. Then, $v_i^N = 0$, and the payoff combination $(0, (\delta v_j^N)_{j \in S_\ell^*(N) \setminus \{i\}})$ is feasible for S; $\sum_{j \in S_\ell^*(N) \setminus \{i\}} \delta v_j^N \leq v(S_\ell^*(N))$. Two cases are possible: (i) If $v_j^N = 0$ for all $j \in S_\ell^*(N) \setminus \{i\}$, then there exists a feasible payoff combination $(y_i, (\delta v_j^N)_{j \in S_\ell^*(N) \setminus \{i\}})$ such that $y_i > 0$ because $v(S_\ell^*(N)) > 0$. (ii) If $v_j^N > 0$ for some $j \in S_\ell^*(N) \setminus \{i\}$, then $\delta v_j^N < v_j^N$. Thus, some $(y_j, (\delta v_j^N)_{j \in S_\ell^*(N) \setminus \{i\}})$, where $y_j > 0$, become feasible solution of the problem (7). Because m^i is the maximum value of (7), we must have $m^i > 0$. Any solution (S, y^S) of (7) satisfies $y_i^S = m^i$ and $y_j^S = \delta v_j^N$ for $j \in S$, $j \neq i$. For any $\varepsilon > 0$, define z^S such that

$$z_i^S = m^i - \varepsilon, \quad z_j^S = y_j^S + \frac{\varepsilon}{|S| - 1}, j \in S, j \neq i.$$

If player i proposes (S, z^S), then it is accepted by all $j \in S$, $j \neq i$. Therefore, $x_i^i \geq z_i^S = m^i - \varepsilon$. Since ε is arbitrary, we conclude $x_i^i \geq m^i$.

Finally, we show that i's proposal is accepted at round 1. It is sufficient to prove $\delta v_i^N < m^i$. Suppose that $\delta v_i^N = m^i$. It follows from $\delta v_i^N \leq v_i^N \leq m^i$ that $m^i = v_i^N = 0$. This contradicts with $m^i > 0$. □

We next present a necessary and sufficient condition for the existence of a pure strategy SSPE of $\Gamma^N(\delta)$. The corresponding theorem in the case of a superadditive game is given in Okada (1996).

Lemma 2. *For $\psi = \{(v^S, \theta^S) \mid S \subset N\}$, where $v^S = (v_i^S)_{i \in S}$, $\theta^S = (T_i^S)_{i \in S}$ and the payoff configuration $\{v^S \mid S \subset N\}$ is feasible in the efficient coalition structure, there exists a pure strategy SSPE σ of $\Gamma^N(\delta)$ with ψ if and only if, for every $S \subset N$ and for every $i \in S$,*

(i) the coalition T_i^S constitutes a solution of

$$\max_{T, y^i} \left(v(T) - \sum_{j \in T, j \neq i} y_j^i \right) \tag{8}$$

$$\text{subject to } y_j^i \geq \delta v_j^N, \text{ for all } j \in S, j \neq i, \text{ and } T \in \mathcal{S}_i.$$

(ii) the expected payoff vector $v^S = (v_i^S)_{i \in S}$ satisfies

$$v_i^S = \frac{1}{|S|} \left\{ v(T_i^S) - \delta \sum_{j \in T_i^S, j \neq i} v_j^S \right\} + \frac{1}{|S|} \sum_{k : i \in T_k^S, k \neq i} v_i^S + \frac{1}{|S|} \delta \sum_{m : i \notin T_m^S} v_i^{S \setminus T_m^S}, \tag{9}$$

where v_i^T is defined to be zero when $T = \emptyset$.

Proof. (only if): Let σ be a SSPE of $\Gamma^N(\delta)$ with $\psi = \{(v^S, \theta^S) \mid S \subset N\}$. We can apply Lemma 1 to every subgame $\Gamma^S(\delta)$. Then, (i) is proved. In the subgame $\Gamma^S(\delta)$, every player i makes a proposal of the payoff allocation $x^i = (x_j^i)_{j \in T_i^S}$ such that

$$x_i^i = v(T_i^S) - \sum_{j \in T_i^S, j \neq i} \delta v_j^S, \quad x_j^i = \delta v_j^S, \ j \in T_i^S, j \neq i. \tag{10}$$

Since this proposal is accepted at round 1, we can obtain (9) by the definition of $\Gamma^S(\delta)$.

(if): Define the strategy combination $\sigma = (\sigma_1, \ldots, \sigma_n)$ of $\Gamma^N(\delta)$ such that, in every subgame $\Gamma^S(\delta)$, every player $i \in S$ proposes a solution (T_i^S, x_i) of the problem (8) satisfying (10), and accepts any proposal (T, y^T) if and only if $y_i^T \geq \delta v_i^S$. It is easy to see that σ is a SSPE of $\Gamma^N(\delta)$ with ψ. □

Let us now turn to the proof of Theorem 2. By the definition of subgame coalitional efficient SSPE, the payoff configuration of such a equilibrium is feasible in the efficient coalition structure. Therefore, we can use Lemma 1 and 2 to prove Theorem 2.

Proof. (only if): Let denote the efficient coalition structure of S by $\pi^*(S) = \{S_1^*(S), \ldots, S_{K^S}^*(S)\}$. Assume that there exists a pure strategy and limit subgame coalitional efficient SSPE of Γ^N exists. It follows from (ii) of Lemma 2 that, for every coalition $S \subset N$ and for every $i \in S^*_\ell(S)$, $\ell = 1, \dots, K^S$,

$$v_i^S = \frac{1}{|S|}\left\{v(S_1^*(S)) - \delta \sum_{k\in S_1^*(S), k\neq i} v_k^S\right\} + \frac{|S_1^*(S)|-1}{|S|}\delta v_i^S$$
$$+ \frac{|S_2^*(S)|}{|S|}\delta v_i^{S\setminus S_2^*(S)} + \cdots + \frac{|S_{K^S}^*(S)|}{|S|}\delta v_i^{S\setminus S_{K^S}^*(S)}, \text{ for all } i \in S_1^*(S),$$
$$\vdots$$
$$v_i^S = \frac{1}{|S|}\left\{v(S_{K^S}^*(S)) - \delta \sum_{k\in S_{K^S}^*(S), k\neq i} v_k^S\right\} + \frac{|S_{K^S}^*(S)|-1}{|S|}\delta v_i^S$$
$$+ \frac{|S_1^*(S)|}{|S|}\delta v_i^{S\setminus S_1^*(S)} + \cdots + \frac{|S_{K^S-1}^*(S)|}{|S|}\delta v_i^{S\setminus S_{K^S-1}^*(S)}, \text{ for all } i \in S_{K^S}^*(S).$$

The above equations system is uniquely solvable for any $\delta < 1$, and the solution $(v_i^S)_{i\in S}$ is expressed in the following recursive form:

(i) For each element of $\pi^*(S)$,

$$v_i^{S_1^*(S)} = \frac{v(S_1^*(S))}{|S_1^*(S)|}, \text{ for all } i \in S_1^*(S),$$
$$\dots, v_i^{S_{K^S}^*(S)} = \frac{v(S_{K^S}^*(S))}{|S_{K^S}^*(S)|}, \text{ for all } i \in S_{K^S}^*(S),$$

(ii) For each union of two elements of $\pi^*(S)$,

$$v_i^{S_1^*(S)\cup S_2^*(S)} = \frac{1}{|S_1^*(S)\cup S_2^*(S)|}\left[v(S_1^*(S)) + |S_2^*(S)|\delta v_i^{S_1^*(S)}\right], \text{ for all } i \in S_1^*(S),$$
$$v_i^{S_1^*(S)\cup S_2^*(S)} = \frac{1}{|S_1^*(S)\cup S_2^*(S)|}\left[v(S_2^*(S)) + |S_1^*(S)|\delta v_i^{S_2^*(S)}\right], \text{ for all } i \in S_2^*(S),$$
$$\vdots$$
$$v_i^{S_{K^S-1}^*(S)\cup S_{K^S}^*(S)} = \frac{1}{|S_{K^S-1}^*(S)\cup S_{K^S}^*(S)|}\left[v(S_{K^S-1}^*(S)) + |S_{K^S}^*(S)|\delta v_i^{S_{K^S-1}^*(S)}\right],$$
$$\text{for all } i \in S_{K^S-1}^*(S),$$
$$v_i^{S_{K^S-1}^*(S)\cup S_{K^S}^*(S)} = \frac{1}{|S_{K^S-1}^*(S)\cup S_{K^S}^*(S)|}\left[v(S_{K^S}^*(S)) + |S_{K^S-1}^*(S)|\delta v_i^{S_{K^S}^*(S)}\right],$$
$$\text{for all } i \in S_{K^S}^*(S).$$

(iii) As a result, we have, for $S = S_1^*(S) \cup \cdots \cup S_{K^S}^*(S)$,

$$v_i^S = \frac{1}{|S|}\left[v(S_1^*(S)) + |S_2^*(S)|\delta v_i^{S\setminus S_2^*(S)} + \cdots + |S_{K^S}^*(S)|\delta v_i^{S\setminus S_{K^S}^*(S)}\right],$$

for all $i \in S_1^*(S)$,

$$\vdots$$

$$v_i^S = \frac{1}{|S|}\left[v(S_{K^S}^*(S)) + |S_1^*(S)|\delta v_i^{S\setminus S_1^*(S)} + \cdots + |S_{K^S-1}^*(S)|\delta v_i^{S\setminus S_{K^S-1}^*(S)}\right],$$

for all $i \in S_{K^S}^*(S)$.

It is easy to see that, for every $S \subset N$ and for the efficient coalition structure $\pi^*(S) = \{S_1^*(S), \ldots, S_{K^S}^*(S)\}$,

$$v_i^S \ \to\ v_i^{S_1^*(S)} = \frac{v(S_1^*(S))}{|S_1^*(S)|}, \text{ for all } i \in S_1^*(S), \tag{11}$$

$$\vdots$$

$$v_i^S \ \to\ v_i^{S_{K^S}^*(S)} = \frac{v(S_{K^S}^*(S))}{|S_{K^S}^*(S)|}, \text{ for all } i \in S_{K^S}^*(S),$$

as δ goes to 1. From (i) of Lemma 2, we have,

for every $i \in S_1^*(S)$,

$$v(S_1^*(S)) - \delta \sum_{k\in S_1^*(S), k\neq i} v_k^S \geq v(T) - \delta \sum_{k\in T, k\neq i} v_k^S \tag{12}$$

for any $T \subset S$ with $i \in T$,

$$\vdots$$

for every $i \in S_{K^S}^*(S)$,

$$v(S_{K^S}^*(S)) - \delta \sum_{k\in S_{K^S}^*(S), k\neq i} v_k^S \geq v(T) - \delta \sum_{k\in T, k\neq i} v_k^S$$

for any $T \subset S$ with $i \in T$.

Taking into account for (11), we have (4) of Theorem 2 as δ goes to 1 in (12).

(if): Suppose that (4) holds. From (ii) of Lemma 2 the expected payoff vector $(v_i^S)_{i\in S}$ satisfies the equations system (9). By the equations system (9) we can easily see that each δv_i^S is monotone increasing with δ and converges to $v(S_\ell^*(S))/|S_\ell^*(S)|$, $i \in S_\ell^*(S)$, $\ell = 1, \ldots, K^S$ as δ goes to 1. Moreover, δv_i^S

is continuous in δ. Therefore, for any δ sufficiently close to 1, we have the following inequalities;

$$v(S_1^*(S)) - \delta \sum_{k \in S_1^*(S), k \neq i} v_k^S \geq \max_{T \in \mathcal{S}_i} \left\{ v(T) - \delta \sum_{k \in T, k \neq i} v_k^S \right\}, \text{ for all } i \in S_1^*(S), \tag{13}$$

$$\vdots$$

$$v(S_{K^S}^*(S)) - \delta \sum_{k \in S_{K^S}^*(S), k \neq i} v_k^S \geq \max_{T \in \mathcal{S}_i} \left\{ v(T) - \delta \sum_{k \in T, k \neq i} v_k^S \right\}, \text{ for all } i \in S_{K^S}^*(S).$$

Let define the strategy combination σ^* of $\Gamma^N(\delta)$ such that, in every subgame $\Gamma^S(\delta)$, every player $i \in S_\ell^*(S)$ proposes the coalition $S_\ell^*(S)$ and the payoff vector y^i such that $y_i^i = v(S_\ell^*(S)) - \sum_{k \in S_\ell^*(S), k \neq i} y_k^i$ and $y_j^i = \delta v_j^S$ for $j \in S_\ell^*(S)$, and accepts any proposal (T, y^T) if and only if $y_i^T \geq \delta v_i^S$. From the above inequalities (13) and Lemma 2, σ^* becomes a SSPE of $\Gamma^N(\delta)$. Thus, as $\delta \to 1$, we have a pure strategy and limit subgame coalitional efficient SSPE of Γ^N with the expected payoff vector (5). □

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