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Political Stability under Asymmetric Information of Powers

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Abstract

We study a political game developed by Acemoglu, Egorov and Sonin (2008) consisting of three players. Each player has an exogenous political power, which is weak or strong. We characterize "politically stable" wealth allocations which have no coalitional deviation to appropriate the wealth of outside players. These allocations are defined as the cores under complete and asymmetric information about their political powers. The core under asymmetric information is a variation of the credible core in Dutta and Vohra (2005). The heterogeneity of players' types produces political stability under complete information and the existence of weak-type players contributes to that under asymmetric information.

Keywords: Political game, asymmetric information of powers, farsighted core, credible core.

JEL codes: C71, D72, D74, D82, H56

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1 Introduction

We study the relationship between a political power distribution and a wealth distribution among players for keeping the political situation to be stable. We consider a three player model. If a political power is considered to be a military power and wealth is considered to be a natural resource in international relations, the stable political situation corresponds to the peaceful situation between the countries. Furthermore, we examine whether asymmetric information about the political powers would decrease or increase in the political stability. Countries in a dispute are usually uninformed about each other's military powers. The question of whether uncertainty about the political powers contributes to peace will be answered. We adopt the core as the concept of stability: this is an allocation with no coalitional deviation. We consider a political game as in Acemoglu, Egorov and Sonin (2008) (henceforth AES). AES considered the formation of a ruling coalition which can take the wealth of all the members of an outside coalition. They showed that an ultimate ruling coalition always exists for any political power distribution and gave an example in which the most powerful player does not necessarily belong to the ultimate ruling coalition. AES were mainly interested in the coalition that ultimately survived. However, we will clarify a condition in which no sub-coalition is formed in the grand coalition. It is assumed that the winning sub-coalition appropriates the wealth of outside players. Therefore, undertaking a coup, a pillage, or a war can be considered forming a sub-coalition. While AES assumed the wealth distribution to be proportional to the distribution of power across players inside the ultimate ruling coalition, we allow any wealth distributions that are independent of the political power distribution. Concretely, we will specify the set of allocations that are not improved upon by a sub-coalition in the grand coalition and examine the effects of asymmetric information about the political powers on the core-stable allocations.

There are a number of previous studies on core allocations in the political game, including Jordan (2006a) and others (Jordan 2006b, 2009; Jung, 2009; and Kerber and Rowat, 2009). Jordan (2006a) described a "pillage game" as a coalitional game in which the power of a coalition depends not only on the size of the coalition but also on the wealth; moreover, any coalition can take the wealth of any less powerful coalition. He defined the core and stable sets of wealth allocation in the pillage game. The core-stable allocation in our paper corresponds to the farsighted core for a consistent expectation (see Jordan, 2006a, p. 42). Moreover, we extend the concept of core to the game with incomplete information. The core in this paper is a variety of the credible core in Dutta and Vohra (2005)[1]. Thus, we consider a situation where private information is transmitted and shared among the members of coalitions in a credible way.

We consider a political game with three players. Each player is endowed with political power, which is strong or weak. A wealth distribution between the players is initially given. We examine a situation where players who have more aggregate political power can appropriate the wealth of a player endowed with less political power. We assume that a single player cannot take away the wealth of other players when all three players are active, but that any coalition with two players has sufficient political power to appropriate the wealth of the outside player.

We obtain the following results on the core-stable wealth allocations under complete and incomplete information. First, when all players are under complete information and have identical political power, whether strong or weak, wealth distributions such that two of the three players share all the wealth equally belong to the core. All other wealth distributions are unstable for coalitional deviations. When two of the three players have the same political

[1]Serrano and Vohra (2007) derived the credible core using the procedure of coalitional voting.

power and the remaining player has a different power under complete information, the core in the political game is given by the set of wealth allocations in which one of the two players having the same political power is endowed with more than half of the aggregate wealth. Thus, the heterogeneity of political powers between players improves the stability of society if the game consists of three players and is under complete information. Second, under incomplete information about their political powers, if all players are strong, the core in the political game with incomplete information is coincident with that with complete information. When two of the three players have strong political power and other player has weak power, the set of core-stable wealth allocations is given by the set of allocations that subtract extremely unequal wealth allocations from the core allocations under complete information. The core under incomplete information is included in the core under complete information. Asymmetry of information about political powers reduces political stability in the case that two of the three players have strong power. Finally, the set of core allocations when two of three players are weak and one player is strong is equal to that when all three players are weak under incomplete information. Roughly speaking, unequal wealth allocations are unstable, while allocations with approximately equal wealth distribution belong to the core in the political game with incomplete information. An equal wealth distribution or a wealth distribution proportional with political power could be stable although these distributions are not stable under complete information. The set of core-stable allocations is enlarged with the number of weak-type players. The existence of weak players contributes to political stability under incomplete information in our setting.

The following studies are related to ours. The model in Piccione and Razin (2005) resembles ours in many ways. They considered power relations over the set of coalitions of players and derived a stable coalition structure with no deviation by forming a new coalition. The objective of each player in their model is to maximize his position in the ranking in society. They focused on a coalition structure that determines the ranking of players, while we consider the stable wealth distribution under a given political power relation. Piccione and Rubinstein (2007) examined a similar situation where stronger countries can take resources costlessly and at will from weaker countries, as we also consider. They studied a exchange economy and defined a "jungle equilibrium" based on a competitive equilibrium.

Our paper is also related to the literature on contests (e.g., Hirshleifer, 1995; Konrad and Skaperdas, 1998; Skaperdas, 1992, 1996, 1998; and Garfinkel and Skaperdas, 2000). These studies deal with conflict between players and countries. The key feature of these studies is that the outcome of conflict is governed by a "contest success function," which specifies the probability of success of each player. The winner in the conflict is stochastically determined. In our model, conflict occurs if two of the three players form a coalition; then, the winner is determined with certainty by their political powers. Skaperdas (1998) examines the problem of coalition formation by using a contest success function. Recently, a dynamic theory of the political game in AES (2008) was provided and developed by Yared (2010), Acemoglu, Egrov and Sonin (2011) and Acemoglu, Golosov, Tsyvinski and Yared (2011). These studies formalize noncooperative extensive form games; in contrast, we examine the core allocation with farsighted expectations of players in a cooperative game. Our paper is also related to models of legislative bargaining under weighted majority voting such as Baron and Ferejohn (1989), Jackson and Moselle (2002) and Snyder, Ting, and Ansolabehere (2008) because we can translate political power into the number of votes. Our approach differs from these studies, however, because we do not consider a specific bargaining procedure and focus on the stable wealth allocation.

The rest of the paper is organized as follows. Section 2 provides the political game with incomplete information. Section 3 examines the core allocations under complete information as a benchmark case. Section 4 defines the concept of the core in a political game with incomplete information and characterizes the core allocations under incomplete information. Section 5 concludes. All proofs are relegated to the Appendix.

2 The Political Game

We consider the political game consisting of three players or countries. Let N denote the set of players and $S \subset N$ denote a coalition. Here, $N = \{1, 2, 3\}$. Each player's type represents the political power of that player. We assume that the type of each player is either weak or strong[2]. The set of types for player i is denoted by $\Theta_i = \{w, s\}$, where $w, s \in \mathbb{R}_{++}$ and $w < s$. $\theta_i \in \Theta_i$ is private information for player i. The type space is $\Theta = \prod_{i \in N} \Theta_i$. Let $q \in \Delta(\Theta)$ denote the prior probability over Θ. Players' types are assumed to be independent. The conditional probability that player i is weak (strong) is $q(w|\theta_{-i}) = p$ ($q(s|\theta_{-i}) = 1 - p$, resp.) for all $\theta_{-i} \in \prod_{j \in N \setminus \{i\}} \Theta_j$ and for all $i \in N$.

A fixed amount of wealth is allocated among players. The total wealth is normalized to unity. The set of wealth allocations is denoted by $X = \{x = (x_i)_{i \in N} \in \mathbb{R}^3_+ | \sum_{j \in N} x_i = 1\}$. The initial wealth allocation $x = (x_i)_{i \in N}$ is commonly known by all players. Players are risk neutral and the payoff function $u_i(x_i)$ for player i is assumed to be x_i.

The power of a coalition S is defined by $\gamma_S = \sum_{j \in S} \theta_j$. When the set of active players is T and the wealth allocation is x, a coalition S is winning in T if $\gamma_S > \gamma_{T \setminus S}$. The members of the winning coalition can take all the wealth

[2] Acemoglu, Egorov and Sonin (2008) allow that each player has an arbitrary political power that is represented by a real number. They ignore a situation where multiple players have the same political power as a generic case. Such a situation occurs with a positive measure in our model.

$\sum_{j \in T \setminus S} x_j$ of the members of $T \setminus S$ and share it equally. Thus, the payoff for player i in the winning coalition S would be $x_i + (\sum_{j \in T \setminus S} x_j)/|S|$. We assume that if $\gamma_S = \gamma_{T \setminus S}$, a coalition S and $T \setminus S$ would win with a probability of 1/2 .

We impose the following assumption about the power relationship between w and s.

Assumption 1. $w < s < 2w$.

Assumption 1 implies that the strong type is not too strong relative to the weak type and that any two-player coalition is always winning in N even if both players are weak. In other words, deviations by a single player always fail when all three players are active.

We introduce the minimum asymmetry of information about the political powers into the model in AES. It is assumed that information about political power becomes public for all players once coalitional deviations occur in the initial state.

3 The Core under Complete Information

In this section, we consider the case where the power of each player is commonly known. The state of the world is given by $(S, (\theta_i)_{i \in S})$, where S is the set of active players and the type profile. Concretely, the initial states of the world are $(N, (s, s, s))$, $(N, (s, s, w))$, $(N, (w, w, s))$, $(N, (w, w, w))$ and its permutations of types between the players.

Let us first consider a complete information case. According to AES, the ultimate ruling coalition is formed starting from the grand coalition $N = \{1, 2, 3\}$. Thus, the elimination process of the outside coalition by the winning coalition continues to be self-enforcing such that none of their sub-coalitions wish to split from the coalition. Every player decides whether to join a coalitional deviation, anticipating the final outcome through the process of elimination by the winning coalitions. The ultimate ruling coalition in this paper corresponds exactly to that in AES. The set of allocations that is immune to any coalitional deviation is called the *farsighted core* in Jordan (2006a)[3].

Let us start with the situation where two players $i, j \in N$ are active. Let $y_i(\{i\}|(\{i,j\},(\theta_i,\theta_j)))$ denote the expected payoff for player i when player i deviates from state $(\{i,j\}|(\theta_i,\theta_j))$. If both players are weak, $(\theta_i,\theta_j) = (w,w)$ (strong, $(\theta_i,\theta_j) = (s,s)$), player i and j win with probability 1/2 by deviating from $\{i,j\}$. If one player is strong and other is weak, the strong player defeats the weak player and obtains all the wealth. Therefore, for all $i, j \in N$,

$$y_i(\{i\}|(\{i,j\},(\theta_i,\theta_j))) = \begin{cases} 0 & \text{if} \quad \theta_i < \theta_j \\ 1/2 & \text{if} \quad \theta_i = \theta_j \\ 1 & \text{if} \quad \theta_i > \theta_j. \end{cases}$$

Applying the same argument, we can see that the above set of allocations corresponds to the core in a political game with two players under complete information.

Next, consider the situation where all three players are active. In this situation, any two-player coalition always wins and takes all the wealth of the outside player. Then, the game restarts from a situation where two players are active. Therefore, players anticipate what happen in the game with the remaining two players when they decide to form a coalition. The expected payoff for player i when coalition $\{i,j\}$ deviates from $\{i,j,k\}$ at state $(\{i,j,k\}|(\theta_i,\theta_j,\theta_k))$ is given by

$$y_i(\{i,j\}|\{i,j,k\},(\theta_i,\theta_j,\theta_k)) = \begin{cases} 0 & \text{if} \quad \theta_i < \theta_j \\ 1/2 & \text{if} \quad \theta_i = \theta_j \\ 1 & \text{if} \quad \theta_i > \theta_j. \end{cases} \tag{1}$$

[3]The concept of farsightedness was introduced by Harsanyi (1974) and further developed by Chwe (1994) and Xue (1998).

Because any deviation by a single player leads to that player being defeated by the rest of the players when three players are active, for all $i \in N$, for all $\theta \in \Theta$, $y_i(\{i\}|\{i,j,k\},(\theta_i,\theta_j,\theta_k)) = 0$.

Definition 1. We say that coalition $T(\subset S)$ *deviates from* $x \in X$ at $(S,(\theta_j)_{j\in S})$ if $y_i(T|(S,(\theta_j)_{j\in S})) > x_i$ for all $i \in T$.

Definition 2. A wealth allocation $x \in X$ is in *the core in the political game* with complete information at $(S,(\theta_j)_{j\in S})$ if there is no sub-coalition T that deviates from x at $(S,(\theta_j)_{j\in S})$.

The following proposition fully characterizes the wealth allocations that belong to the core in the political game with complete information when three players are active.

Proposition 1. *(i) If* $\theta = (w,w,w)$ *or* $\theta = (s,s,s)$*, the core in the political game with complete information at* $(\{1,2,3\},\theta)$ *is* $\{(1/2,1/2,0),(1/2,0,1/2),(0,1/2,1/2)\}$.
(ii) If $\theta = (\theta_i,\theta_j,\theta_k) = (w,w,s)$ *or* $\theta = (\theta_i,\theta_j,\theta_k) = (s,s,w)$*, the core in the political game with complete information at* $(\{i,j,k\},\theta)$*, where* $i,j,k = 1,2,3, i \neq j \neq k$*, is* $\{x \in X \mid (x_i + x_j + x_k = 1)$ *and* $((x_i \geq 1/2)$ *or* $(x_j \geq 1/2))\}$.

Figure 1 describes the core allocations for (i) and (ii) in Proposition 1.

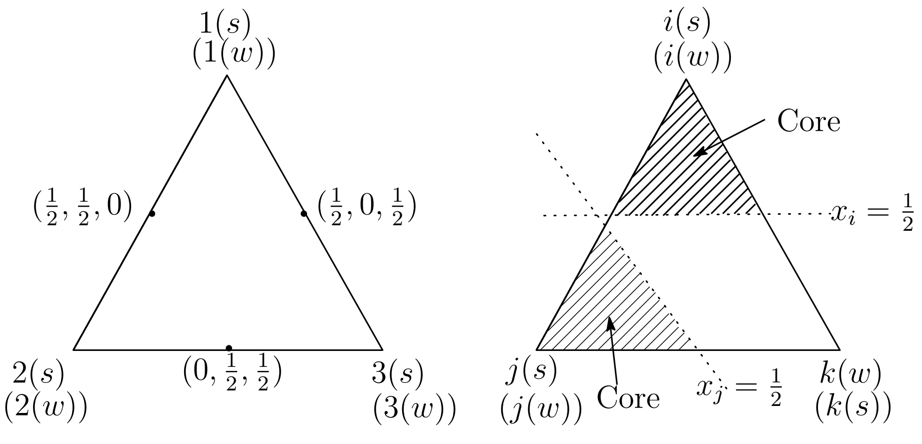

Figure 1: Core under complete information

The core allocation in the political game at $(N,(w,w,w))$ and $(N,(s,s,s))$ is given by an allocation in which two of the three players share the aggregate wealth equally. The set of core allocations at $(N,(w,w,s))$ and $(N,(s,s,w))$ is the set of allocations in which one of the two players of the same type obtain wealth greater than or equal to half of the aggregate wealth. The set of core allocations at $(N,(w,w,w))$ and $(N,(s,s,s))$ is a proper subset of that at $(\{i,j,k\}(w,w,s))$ and $(\{i,j,k\}(s,s,w))$. In the three-player political game, the heterogeneity of players' types produces political stability. This result may be specific to the political game with three players since the inclusion property does not hold in the two-player game.

4 The Core under Incomplete Information

4.1 Transmission and sharing of information

In this section, we consider the political game with incomplete information.

For an event $E \subset \Theta$, let E_i denote the corresponding set of types for player i, that is, $E_i = \{\theta_i \in \Theta_i | \theta \in E\}$. For E, let us define $V_i(E) = \Theta_i \setminus E_i$. The probability that player i assigns to $\theta_{-i} \in \Theta_{-i}$, conditional on her type being θ_i and her belief that the true state lies in E, is defined by

$$q(\theta_{-i}|\theta_i, E) = \frac{q(\theta_i, \theta_{-i})}{\sum_{\hat{\theta}_{-i} \in E_{-i}} q(\theta_i, \hat{\theta}_{-i})}.$$

For the political game with incomplete information at the realized state $(\{i,j,k\},(\theta_i,\theta_j,\theta_k))$, the conditional expected payoff for player i when a two-player coalition $\{i,j\}$ deviates from the grand coalition N and the members of coalition $\{i,j\}$ believe that the true state is in an event E is given by

$$Y_i(\{i,j\}|(\{i,j,k\},(\theta_i,\theta_j,\theta_k)),E) = \sum_{\hat{\theta}_{-i} \in E_{-i}} q(\hat{\theta}_{-i}|\theta_i,E) y_i(\{i,j\}|(\{i,j,k\},(\theta_i,\hat{\theta}_{-i}))).$$

Before defining the core concept under incomplete information, we examine the information transmission and information sharing among the members of coalitions in the political game. Here, the information about the players' types is transmitted through the approval of players to join a coalition given a wealth distribution. If there were no information transmission by forming a two-player coalition, every member of the coalition would have a prior belief that the partner is a strong type with probability p.

Consider a weak-type player having a prior belief. Under Assumption 1, if a two-player coalition is formed, the coalition defeats the outside player with probability one. After the deviation, the incompleteness of information about players' types is resolved. Only two players are active; then, two cases can occur. First, if the partner is the strong type, the weak-type player loses and obtains a payoff of 0. Second, if the partner is the weak type, he obtains the expected payoff of 1/2 because his winning probability is 1/2. In the initial state, a player with a prior belief evaluates the former case with probability p and the latter case with probability $1-p$. Thus, a weak-type player with a prior belief anticipates the expected payoff of $p \times 0 + (1-p)(1/2) = (1-p)(1/2)$ by a deviation of forming a two-player coalition. Therefore, the weak-type player participates in forming a two-player coalition if and only if his wealth level x_i is less than $(1-p)(1/2)$.

Next consider a strong-type player with a prior belief. Any two-player coalition wins when all three players are active. The strong player expects that the partner is the strong type with probability p and the weak type with probability $1-p$. If the partner is the strong type, the strong player obtains the expected payoff of 1/2 because the winning probability is 1/2. If the partner is the weak type, the strong player obtains a payoff of one. Thus, a strong-type player with a prior belief about the partner's type obtains the expected payoff of $p \times (1/2) + (1-p) \times 1 (= 1 - p/2)$ by agreeing to form a two-player coalition. Therefore, the strong-type player agrees to form a two-player coalition if and only if his wealth level x_i is less than $1 - p/2$.

From the above behaviors of a weak-type and a strong-type players, we conclude that the following information is transmitted by the approval of players for joining a coalition at the initial state. If $0 \leq x_i < (1/2) - (p/2)$, then player i joins a two-player coalition whether he is the strong or weak type. An agreement to join the coalition does not transmit any information and, therefore, the partner of player i has a prior belief about player i's type as before. If $(1/2) - (p/2) \leq x_i < 1 - (p/2)$, only a player of the strong type joins the coalition, but a player of the weak type does not. Therefore, the information that player i is the strong type is revealed to the partner. If $x_i \geq 1 - (p/2)$, neither a player of a strong type nor a player of a weak type join to form a coalition under a prior belief.

Note that the information that a player is a weak type could not be transmitted in the political game. Under the updated belief that the partner is a strong type with probability one, a weak-type player with any wealth level x_i does not join the coalition because his expected payoff is zero. On the other hand, a strong-type player joins the coalition if and only if his wealth x_i is less than 1/2.

4.2 Credible core

We extend the core concept in the previous section to the political game with incomplete information.

We consider the following coalitional voting game as in Serrano and Vohra (2007). The initial wealth allocation $x \in X$ is given. Coalition S considers whether to deviate from x. The members of S vote to either accept or reject the deviation. If all the members accept the deviation, coalition S is formed. Then, informational asymmetry about types is resolved; that is, all players' types become a public information and coalition S then fights against coalition $N \setminus S$. Subsequently, the political game under complete information is played until the ultimate ruling coalition is achieved. As Serrano and Vohra (2007) have shown, the credible core in Dutta and Vohra (2005) is derived as the set of allocations that no coalition would vote to give up in favor of some other feasible allocation. However, in the credible core, the transmission of private information between the members of a coalition is permitted in the following sense.

Definition 3. Given a true type profile $\theta \in \Theta$, for a sub-coalition S and event E satisfying $\theta \in E$, an allocation $(Y_i)_{i \in S} \in A_S$ for the members of coalition S is said to satisfy *self-selection* with respect to x over E at (N, θ) if

$$Y_i(S|(N, (\hat{\theta}_i, \theta_{-i})), E) \leq x_i \quad \text{for all } \hat{\theta}_i \in V_i(E) \quad \text{for all } i \in S. \quad \text{(SS)}$$

Definition 4. Given a true type profile $\theta \in \Theta$, for a sub-coalition S and event E satisfying $\theta \in E$, an allocation $(Y_i)_{i \in S} \in A_S$ *dominates* x over E at (N, θ) if

$$Y_i(S|(N, (\theta_i, \theta_{-i})), E) > x_i \quad \text{for all } \theta_i \in E \quad \text{for all } i \in S. \quad \text{(D)}$$

Definition 5. Coalition S is said to have a *credible deviation* to x at (N, θ) if there exists an allocation $(Y_i)_{i \in S}$ for S and an event E such that (SS) and (D) are satisfied at (N, θ). The *credible core* at (N, θ) consists of all allocations to which there does not exist a credible deviation at (N, θ).

Our definition of the credible core is different from that in Dutta and Vohra (2005) on some points. First, we consider the coalitional stability for a wealth allocation that is realized at the initial state, while Dutta and Vohra consider for an incentive compatible (direct revelation) mechanism. Therefore, the incentive compatibility condition is dropped here. Moreover, asymmetric information is inherent and persistent in Dutta and Vohra (2005), but it is resolved after any coalitional deviation in our model.

4.3 Characterization of credible core

We characterize the core allocation in the political game with incomplete information. The core allocations depend on which state is realized.

When a type profile $(\theta_1, \theta_2, \theta_3) = (s, s, s)$ is selected by nature, the core in the political game under incomplete information is coincident with that under complete information. See Figure 2.

Proposition 2. *At state $(N, (s, s, s))$, the credible core in the political game with incomplete information consists of* $(1/2, 1/2, 0)$, $(1/2, 0, 1/2)$ *and* $(0, 1/2, 1/2)$.

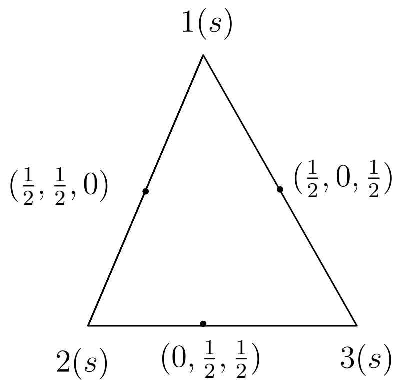

Figure 2: The credible core at $(N, (s, s, s))$

Next, consider state $(\{i, j, k\}, (s, s, w))$. Players i and j are the strong type and player k is the weak type, where $i, j, k \in \{1, 2, 3\}$, $i \neq j \neq k$. The initial wealth allocation is given by $x = (x_i, x_j, x_k)$.

Proposition 3. *At state $(\{i, j, k\}, (s, s, w))$, the credible core in the political game with incomplete information consists of*

$$\left\{ x \in X \,\middle|\, \frac{1}{2} \leq x_i \leq \frac{1}{2} + \frac{p}{2} \text{ and } \frac{1}{2} - \frac{p}{2} \leq x_j \leq \frac{1}{2} \right\}, \tag{2}$$

$$\left\{ x \in X \,\middle|\, \frac{1}{2} \leq x_i \leq \frac{1}{2} + \frac{p}{2} \text{ and } \frac{1}{2} - \frac{p}{2} \leq x_k \leq \frac{1}{2} \right\}, \tag{3}$$

$$\left\{ x \in X \,\middle|\, \frac{1}{2} - \frac{p}{2} \leq x_i \leq \frac{1}{2} \text{ and } \frac{1}{2} \leq x_j \leq \frac{1}{2} + \frac{p}{2} \right\}, \tag{4}$$

$$\left\{ x \in X \,\middle|\, \frac{1}{2} \leq x_j \leq \frac{1}{2} + \frac{p}{2} \text{ and } \frac{1}{2} - \frac{p}{2} \leq x_k \leq \frac{1}{2} \right\}. \tag{5}$$

The shaded region in Figure 3 indicates the credible core in the political game with incomplete information. The core with incomplete information is

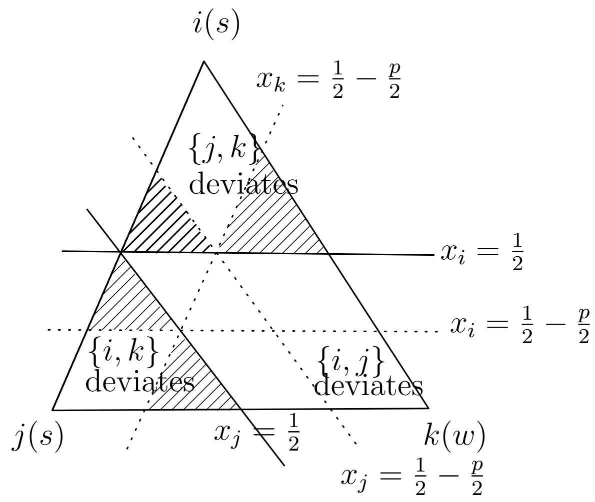

Figure 3: The credible core at $(N, (s, s, w))$

a proper subset of the core with complete information at $(N, (s, s, w))$. This implies that the informational asymmetry reduces the political stability at these states. The set of the credible core is enlarged with p and converges to the core with complete information at (s, s, w) as $p \to 1$.

Given p, the credible cores in the political game with incomplete information at states $(N, (w, w, s))$ and $(N, (w, w, w))$ are same. At state $(N, (w, w, s))$, let us call the weak-type players i and j and the strong-type player k, where $i, j, k = 1, 2, 3$, $i \neq j \neq k$.

Proposition 4. *At states $(\{i, j, k\}, (w, w, s))$ and $(\{i, j, k\}, (w, w, w))$, the credible core in the political game with incomplete information is*

$$X \backslash \Bigg\{ \left\{ x \in X \middle| 0 \leq x_i < \frac{1}{2} - \frac{p}{2} \text{ and } 0 \leq x_j < \frac{1}{2} - \frac{p}{2} \right\}$$
$$\bigcup \left\{ x \in X \middle| 0 \leq x_i < \frac{1}{2} - \frac{p}{2} \text{ and } 0 \leq x_k < \frac{1}{2} - \frac{p}{2} \right\}$$
$$\bigcup \left\{ x \in X \middle| 0 \leq x_i < \frac{1}{2} - \frac{p}{2} \text{ and } 0 \leq x_j < \frac{1}{2} - \frac{p}{2} \right\} \Bigg\}.$$

There is no set inclusion relation between the credible core with incomplete information and that with complete information at $(\{i, j, k\}, (w, w, s))$ if $p < 1$. Wealth allocations satisfying (i) $x_j > 1/2 - p/2$, $x_k > 1/2 - p/2$ and $x_i \geq 1/2$

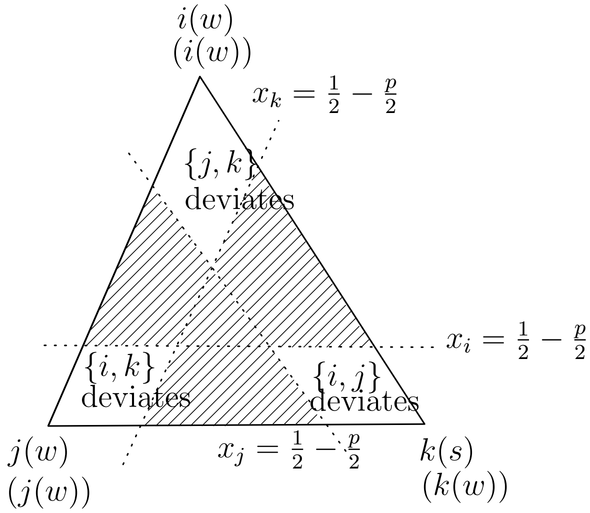

Figure 4: The credible core at $(\{i, jk\}, (w, w, s))$ and $(\{i, j, k\}, (w, w, w))$

and (ii) $x_i > 1/2 - p/2$, $x_k > 1/2 - p/2$ and $x_j \geq 1/2$ are not in the core with incomplete information but are in the core with complete information. On the other hand, wealth allocations satisfying $1/2 - p/2 < x_i < 1/2$ and $1/2 - p/2 < x_j < 1/2$ are changed to be the core allocation under incomplete information. The effect of asymmetric information on political stability is ambiguous. Unequal wealth distributions are unstable, but equal wealth distributions are stable under incomplete information. In contrast, the credible core with incomplete information contains the core with complete information at $(N, (w, w, w))$. Asymmetric information leads to political stability at state $(N, (w, w, w))$.

At states $(N, (w, w, s))$ and $(N, (w, w, w))$, the credible core converges to the set of all wealth allocations as $p \to 1$. However, $p \to 1$ implies that the *ex ante* probability that state $(N, (w, w, s))$ or $(N, (w, w, w))$ occurs is almost zero. Comparing the credible cores at $(N, (s, s, s))$, $(N, (s, s, w))$, $(N, (w, w, s))$ and $(N, (w, w, w))$, we conclude that the core allocations are enlarged with the number of weak-type players. The existence of players of the weak type contributes to political stability under incomplete information.

5 Conclusion

We studied a stable wealth distribution such that there is no coalition formation with the goal of a pillage of wealth under a given political power distribution among three players. We compared the results for political power with complete information with that for power with asymmetric information about them. Our results on the core-stable wealth allocations depend on the initial wealth distribution, the combinations of political powers between players, and the number of weak- or strong-type players. It is difficult to obtain qualitative and generalized results. We now discuss some extensions of our model.

The first is a relaxation of Assumption 1. If Assumption 1 is not satisfied, players of the strong type become more aggressive while players of the weak type become more conservative. Furthermore, only one player of the strong type may win against the other two players in the initial state. There is a case in which a coalition of two weak-type players loses, in contrast with our setting. Therefore, we guess that in this case, the set of core allocations where all players are weak is enlarged and that where all players are strong becomes smaller. However, the effect on political stability when two types of players coexists is ambiguous.

The second is an extension to an n-player game. It is easy to see that in a political game with two players, both sets of core allocations at states $(N, (s, s))$ and $(N, (s, w))$ are empty. A core-stable allocation at state $(N, (w, w))$ is given by $(1/2, 1/2)$. The relationship to our results in a three-player case is not clear. The number of players significantly affects the core in the political game with incomplete information as in the case of majority voting. Therefore, we should extend the model to an n-player game, paying attention to the number of players.

Appendix

Proof of Proposition 1

(i) Consider the game at state $(N, (w, w, w))$.

Let us show that allocations $(x_1, x_2, x_3) = (1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2)$ are in the core. These are allocations in which two of the three players obtain 1/2. We denote them by $(x_i, x_j, x_k) = (1/2, 1/2, 0)$. First, consider a deviation by coalition $\{i, j\}$. By (1), we have that

$$y_i(\{i,j\}|(N,(w,w,w))) = \frac{1}{2},$$
$$y_j(\{i,j\}|(N,(w,w,w))) = \frac{1}{2}.$$

Because

$$x_i = \frac{1}{2} = y_i(\{i,j\}|(N,(w,w,w))),$$
$$x_j = \frac{1}{2} = y_j(\{i,j\}|(N,(w,w,w))),$$

coalition $\{i, j\}$ does not deviate from (x_i, x_j, x_k). Next, consider a coalitional deviation by $\{i, k\}$. Because

$$y_i(\{i,k\}|(N,(w,w,w))) = \frac{1}{2},$$
$$y_k(\{i,k\}|(N,(w,w,w))) = \frac{1}{2},$$

player i does not joint coalition $\{i, k\}$ because $y_i(\{i, k\}|(N, (w, w, w))) = 1/2 = x_i$, while player k have an incentive to join coalition $\{i, k\}$. Therefore, coalition $\{i, k\}$ cannot be formed under (x_i, x_j, x_k). Finally, consider a deviation by coalition $\{j, k\}$. Because player i and j are symmetric, we can apply the same argument to coalition $\{j, k\}$ as coalition $\{i, k\}$. As a result, there is no coalition which deviates from x. Thus, allocation $(x_i, x_j, x_k) = (1/2, 1/2, 0)$ is in the core.

Second, let us show that any allocation except $(x_i, x_j, x_k) = (1/2, 1/2, 0)$ does not belong to the core. If $x = (x_1 x_2, x_3) \neq (1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2)$, then there exists $\{i, j\} \subset N$ such that $x_i < 1/2$ and $x_j < 1/2$. Because

$$y_i(\{i,j\}|(N,(w,w,w))) = \frac{1}{2} > x_i,$$
$$y_j(\{i,j\}|(N,(w,w,w))) = \frac{1}{2} > x_j,$$

coalition $\{i, j\}$ deviates from x at $(N, (w, w, w))$. This implies that x is not in the core.

We can apply the same argument to state $(N, (s, s, s))$ as state $(N, (w, w, w))$. Therefore, only $(x_i, x_j, x_k) = (1/2, 1/2, 0)$ is in the core in the game with complete information at $(N, (s, s, s))$.

(ii) Consider the game at state $(\{i, j, k\}, (s, s, w))$, where $i, j, k \in \{1, 2, 3\}$, $i \neq j \neq k$. Two of the three players, players i and j, are the strong type s.

Let us show that any allocation such that $x_i + x_j + x_k = 1$ and $x_i \geq 1/2$ is in the core in a political game with complete information. Since $x_i \geq 1/2$, $x_j + x_k < 1/2$. Moreover, $x_j < 1/2$ and $x_k < 1/2$. Because

$$y_i(\{i,j\}|(\{i,j,k\},(s,s,w))) = \frac{1}{2} \leq x_i,$$

player i never join coalition $\{i, j\}$ to deviate from x. Therefore, coalition $\{i, j\}$ is not formed because player i and j must agree unanimously to form $\{i, j\}$. Next, consider formations of coalition $\{i, k\}$ and $\{j, k\}$. Because player i is the weak type, we have that

$$y_k(\{i,k\}|(\{i,j,k\},(s,s,w))) = y_k(\{j,k\}|(\{i,j,k\},(s,s,w))) = 0 \leq x_k.$$

Thus, coalitions $\{i, k\}$ and $\{j, k\}$ do not deviate from x. There is no coalitional deviation from x. This implies that $x \in X$ such that $x_i + x_j + x_k = 1$ and $x_i \geq 1/2$ is in the core.

We can prove that an allocation $(x_i, x_j, x_k) \in X$ satisfying $x_i + x_j + x_k = 1$ and $x_j \geq 1/2$ is in the core in a same way because players i and j are symmetric.

Finally, consider the rest of the allocations; that is, $\hat{X} = X \setminus \{x \in X|(x_i \geq 1/2) \text{ or } (x_j \geq 1/2)\}$. Let $\hat{x} = (\hat{x}_i, \hat{x}_j, \hat{x}_k) \in \hat{X}$. These allocations satisfy that

$\hat{x}_i < 1/2$ and $\hat{x}_j < 1/2$. Because players i and j are strong, both players obtain the expected payoff of 1/2 through forming coalition $\{i, j\}$. Then, we have that

$$y_i(\{i,j\}|(\{i,j,k\},(s,s,w))) = \frac{1}{2} > \hat{x}_i,$$
$$y_j(\{i,j\}|(\{i,j,k\},(s,s,w))) = \frac{1}{2} > \hat{x}_j.$$

Thus, coalition $\{i, j\}$ deviates from $\hat{x}$.

For a political game at state $(\{i,j,k\},(w,w,s))$, the same procedure as in state $(\{i,j,k\},(s,s,w))$ can be applied to prove that $\{x \in X|(x_i \geq 1/2)$ or $(x_j \geq 1/2)\}$ is the core. Therefore, we omit the proof.

Proof of Proposition 2

(i) Let us show that $(x_i, x_j, x_k) = (1/2, 1/2, 0)$, where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$, is in the credible core. Note that $x_i = 1/2 \in [1/2 - p/2, 1 - p/2)$ and $x_j = 1/2 \in [1/2 - p/2, 1 - p/2)$. Therefore, the information that player i and j are the strong type is transmitted among the members of coalitions if they agree to join the coalitional deviations. First, consider a deviation by coalition $\{i, j\}$. In this case, both players have commonly known that they are the strong type. The event E for coalition $\{i, j\}$ is given by $E = E_i \times E_j \times E_k = \{s\} \times \{s\} \times \Theta_k$. Because $q(s, s|s, E) = p$, $q(s, w|s, E) = 1 - p$ and $y_i(\{i,j\}|(\{i,j,k\},(s,s,s)) = 1/2$, $y_i(\{i,j\}|(\{i,j,k\},(s,s,w))) = 1/2$, we have that

$$Y_i(\{i,j\}|(\{i,j,k\},(s,s,s)),E) = \frac{1}{2}.$$

Player i does not join coalition $\{i, j\}$ because $Y_i(\{i,j\}|(\{i,j,k\},(s,s,s)),E) = 1/2 = x_i$. Player j also does not join coalition $\{i, j\}$ for the same reason.

Next, consider a deviation by coalition $\{i, k\}$. Player k believes that player i is the strong type with probability one. If player k is the weak type, he does not join the deviation. Then, players i and k have a common belief that player k is also the strong type if he joins coalition $\{i, k\}$. Under their beliefs, players i

and k expect the payoff of 1/2 through the deviation by coalition $\{i, k\}$. Thus, $Y_i(\{i, k\}|(\{i, j, k\}, (s, s, s)), \{s\} \times \Theta_j \times \{s\}) = 1/2$. Because $x_i = 1/2$, player i would not join the coalitional deviation. A deviation by coalition $\{i, k\}$ cannot be agreed. We can apply the same argument to coalition $\{j, k\}$ as coalition $\{i, k\}$. Because no coalitional deviation exists, $(x_i, x_j, x_k) = (1/2, 1/2, 0)$ is in the credible core.

(ii) We show that any allocation $x \neq (1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2)$ is not in the credible core. For x, there are player i and j such that $0 \leq x_i < 1/2$ and $0 \leq x_j < 1/2$. If $0 \leq x_i < 1/2 - p/2$ and $0 \leq x_j < 1/2 - p/2$, there is no information transmission among the members of coalition $\{i, j\}$. Suppose that $E = \Theta_i \times \Theta_j \times \Theta_k$. Because

$$
\begin{aligned}
Y_i(\{i, j\}|(N, (s, s, s)), E) &= \frac{1}{2} + \frac{p}{2} > x_i, \\
Y_j(\{i, j\}|(N, (s, s, s)), E) &= \frac{1}{2} + \frac{p}{2} > x_j,
\end{aligned}
$$

coalition $\{i, j\}$ deviates from x.

If $1/2 - p/2 \leq x_i < 1/2$ and $1/2 - p/2 \leq x_j < 1/2$, the information that player i and j are the strong type is commonly known by the members of coalition $\{i, j\}$. Choose $E = \{s\} \times \{s\} \times \Theta_k$. Because

$$
\begin{aligned}
Y_i(\{i, j\}|(\{i, j, k\}, (w, s, s)), E) = 0 < x_i, \\
Y_j(\{i, j\}|(\{i, j, k\}, (s, w, s)), E) = 0 < x_j,
\end{aligned}
$$

an allocation $(Y_\ell)_{\ell \in \{i,j\}}$ satisfy (SS) with respect to x over E. Moreover, we have that

$$
\begin{aligned}
Y_i(\{i, j\}|(\{i, j, k\}, (s, s, s)), E) &= \frac{1}{2} > x_i, \\
Y_j(\{i, j\}|(\{i, j, k\}, (s, s, s)), E) &= \frac{1}{2} > x_j.
\end{aligned}
$$

Therefore, an allocation $(Y_\ell)_{\ell \in \{i,j\}}$ dominates x over E. Thus, coalition $\{i, j\}$ has a credible deviation to x.

If $1/2 - p/2 \leq x_i < 1/2$ and $0 \leq x_j < 1/2 - p/2$, the information that player i is the strong type is transmitted. Because

$$Y_i(\{i,j\}|(\{i,j,k\},(w,s,s)),\{s\} \times \Theta_j \times \Theta_k) = \frac{1}{2} - \frac{p}{2} \leq x_i,$$

the self-selection condition is satisfied over $E = \{s\} \times \Theta_j \times \Theta_k$. Furthermore, because

$$Y_i(\{i,j\}|(\{i,j,k\},(s,s,s)),\{s\} \times \Theta_j \times \Theta_k) = \frac{1}{2} + \frac{p}{2} > x_i,$$
$$Y_j(\{i,j\}|(\{i,j,k\},(s,s,s)),\{s\} \times \Theta_j \times \Theta_k) = \frac{1}{2} > x_j,$$

an allocation $(Y_\ell)_{\ell \in \{i,j\}}$ dominates x over $E = \{s\} \times \Theta_j \times \Theta_k$. Thus, coalition $\{i,j\}$ has a credible deviation to x.

Proof of Proposition 3

(i) We first show that the set of allocations represented by (2) is in the core. Because $x_i \geq 1/2 - p/2$ and $x_j \geq 1/2 - p/2$ in (2), it is commonly known that player i and j are the strong type if they join coalitional deviations. Therefore, player k of the weak type does not join coalition $\{i,k\}$ and $\{j,k\}$ because his expected payoff by these coalitional deviations are zero. When coalition $\{i,j\}$ is formed, player i and player j have the expected payoff of $1/2$ because they believe that the opponent is the strong type with probability one. Because player i has his wealth of $x_i \geq 1/2$, he does not agree to form coalition $\{i,j\}$. Because there is no coalitional deviation, allocations in (2) are in the credible core.

(ii) Next let us show that allocations in (3) are in the credible core. Player k of the weak type does not join any coalitional deviation because he has already obtained $x_i \geq 1/2 - p/2$. Only a deviation by coalition $\{i,j\}$ is possible. However, player i does not agree to join the coalition because $x_i \geq 1/2$. We conclude that allocations in (3) are in the core.

Furthermore, we can apply the same argument to allocations in (4) and (5) as in (2) and (3) if interchanging player i for player j

(iii) It is proved that $\{x \in X | x_j < 1/2 - p/2 \text{ and } x_k < 1/2 - p/2\}$ and $\{x \in X | x_i < 1/2 - p/2 \text{ and } x_k < 1/2 - p/2\}$ are not in the credible core in a same manner. Consider a deviation by coalition $\{j, k\}$. No information about their types is transmitted between player j and k in this case. Because

$$Y_j(\{j,k\}|(\{i,j,k\},(s,s,w)),\Theta_i \times \Theta_j \times \Theta_k) = \frac{1}{2} + \frac{p}{2} > x_j,$$
$$Y_k(\{j,k\}|(\{i,j,k\},(s,s,w)),\Theta_i \times \Theta_j \times \Theta_k) = \frac{1}{2} - \frac{p}{2} > x_k,$$

coalition $\{j, k\}$ is formed.

(iv) If repeating part (ii) in the proof of Proposition 2, we can show that $\{x \in X | x_i < 1/2 \text{ and } x_j < 1/2\}$ are not the credible core because players i and j are the strong type.

Proof of Proposition 4

We can prove that the sets of $\{x \in X | x_i < 1/2 - p/2 \text{ and } x_j < 1/2 - p/2\}$, $\{x \in X | x_i < 1/2 - p/2 \text{ and } x_k < 1/2 - p/2\}$ and $\{x \in X | x_j < 1/2 - p/2 \text{ and } x_k < 1/2 - p/2\}$ are not in the core allocations by applying the same argument as (iii) in the proof of Proposition 3.

For any allocation except for the above allocations, there are at least two players i and j whose wealth allocations x_i and x_j are greater than or equal to $1/2 - p/2$. Because a player of the weak type does not join a coalition if his wealth allocation is greater than or equal to $1/2 - p/2$, any coalition cannot be formed. Thus, these allocations are in the credible core.

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