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Inequality and a multiple subgroup-decomposition method

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abstract

The Oaxaca-Blinder decomposition method is a popular and useful tool for analyzing multiple factors for inequality and its change. However, this method may suffer from regression errors such as the endogeneity problem and the questionable assumptions. The other popular tool for decomposing inequality and its change is subgroup decomposition. However, this method can identify not multiple factors but a single factor for inequality and its change.

This paper proposes a multiple subgroup-decomposition method which can identify multiple factors for inequality and its changes without estimating wage or earning equations. An empirical application of this method is implemented to analyze the Japanese wage inequality for full-time workers on the basis of a micro-level data set obtained from the Employment Status Survey (1992-2002).

It is found that wage inequality had a constant or slightly decreasing trend from 1992 to 1997, however, wage inequality increased from 1997 to 2002. The changes in overall wage inequality are partly due to the decreasing effect of between-age-group inequality and between-education-group inequality and increasing effect of between-firm-size-group inequality both from 1992 to 1997 and from 1997 to 2002. These decreasing effects of between-age-group inequality could be identified by the multiple subgroup-decomposition method, however, which could not identified by a traditional single subgroup-decomposition. Therefore, practitioners should use the multiple decomposition method when implementing the inequality decomposition precisely.

Further, it is found that the factors for the expanding overall inequality from 1997 to 2002 are mainly attributed to the increase in within-group inequality for groups of identical age, education, and firm size. The increase may be caused by the extensive introduction of performance pay in the late 1990s.

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JEL Classification Numbers: C81; D31; D63; I24; J21; J31

1 Introduction

Many studies have attempted to explain the factors for inequality and its change using various inequality decomposition methods. There are distinct two decomposition method that can analyze inequality factors (Cowell and Fiorio 2011).

One is the method broadly called the Oaxaca-Blinder (OB) decomposition method, which employes regression analysis. This method uses the estimated results of wage or earning equations and can decompose the difference in a distributional statistic between two groups or its change over time into various explanatory factors. The OB decomposition method is a popular and useful tool for analyzing the factors for changes in wage inequality and distribution (Fortin et al. 2011).

The other method is often called \textit{a priori} approaches such as the subgroup-decomposition by population subgroups in Mookherjee and Shorrocks (1982) and factor-source decomposition in Shorrocks (1982).\textsuperscript{1} Although the OB decomposition method has an advantage in that it can identify multiple factors for changes in inequality, it may suffer from regression errors such as the endogeneity problem. The (single) subgroup-decomposition method can easily identify the single factor for changes in inequality; however, it cannot identify multiple factors while controlling other factors.

The main purpose of this paper is to present a subgroup decomposition method which can implement the decomposition analysis controlling multiple factors and identify multiple factors for the change in inequality. Mookherjee and Shorrocks (1982) mentioned that simultaneous multiple subgroup-decomposition is, in principle, feasible, however, they did not show a mathematical background or an empirical application.

The multiple subgroup-decomposition method proposed in this paper has a number of strengths. It does not need to estimate the regression of the wage equation, and therefore, regression errors do not occur. Moreover, it can easily decompose the inequality just as in Mookherjee and Shorrocks (1982). The paper also presents an empirical application of this method to examine the factors for the changes in Japanese wage inequality from 1992 to 2002, on the basis of the Employment Status Survey (ESS) data. In the empirical application, this paper found that a single subgroup decomposition method had a limitation to identify the effect of the factor for inequality change precisely, and the new method proposed is useful for discerning the specific effects of the factor.\textsuperscript{2}

\textsuperscript{1}The subgroup-decomposition method was shown by Bouguignon (1979), Cowell (1980) and Shorrocks (1980, 1984).

\textsuperscript{2}This paper does not make a comparison between our method and the Shapley decomposition method proposed by Shorrocks (2013).
The paper is organized as follows. In section 2, I provide an overview of the decomposition methods and exposit subgroup-decomposition in detail using the mean log deviation (MLD) presented by Mookherjee and Shorrocks (1982). In addition, I show a multiple subgroup-decomposition method that can simultaneously decompose two or more factors for changes in wage inequality. Section 3 presents an empirical application of the method proposed to the Japanese wage inequality among full-time workers between 1992 and 2002; this highlights the usefulness of the multiple subgroup-decomposition. Section 4 concludes the paper.

2 The method

In this section, I review the two main strands of inequality decomposition methods, OB decomposition and subgroup decomposition. I give a detailed account of subgroup decomposition before proposing an extending method that can simultaneously decompose two or more factors for changes in an inequality.

2.1 Overview of decomposition methods

The Oaxaca-Blinder (OB) decomposition method was developed in the 1970s to analyze black-white and male-female wage differentials. Though, the Blinder and Oaxaca’s original works (1973) analyzed the mean wage differential of each group, but since the 1990s, the OB decomposition method has been further developed and used to analyze the decomposition of distributional inequality statistics such as variance, the Gini coefficient and percentiles (e.g., DiNardo et al. 1996, Card and DiNardo 2002, Lemieux 2006, Kambayashi et al. 2008, Bourguignon et al. 2008, Firpo et al. 2009, Lemieux et al. 2009). Using regression parameters, covariance matrices, and residual estimated from the wage equation, the change in variance over two periods can be decomposed into (i) changes in the wage structure that are captured by the changes in the estimated coefficients of the wage equation, (ii) the changes in the variance of workers’ attributes, which are captured by the change in the variance of the explanatory variables of the wage equation, and (iii) the changes in the variance of the error term of the wage equation. Fortin et al. (2011) conducted a comprehensive overview of OB decomposition methods. The OB decomposition method has some advantages. Practitioners can simultaneously and efficiently extract many factors behind changes in income distributions and inequality, by choosing the explanatory variables of the regression such as gender, age, experience, education, firm size, and the influences of minimum wage and union density.4

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3 This paper does not focus on the factor source decomposition method.
4 Fields (2003) and Cowell and Fiorio (2011) show that inequality level can decompose each attribute using the estimation results of wage or income equation.
However, the OB decomposition method has two drawbacks. One is complication of the estimation, which includes questionable assumptions (Cowell and Fiorio 2011) and the occurrence of regression errors caused by the endogeneity problem. The other disadvantage is the omitted (reference) group problem. The changes in the estimated coefficients in the term in decomposition arbitrarily depend on the choice of the categorical covariates in the regression; this was demonstrated by Oaxaca and Ransom (1999), who explained that conventional decomposition methodology cannot identify the separate contributions of dummy variables to the wage decomposition because it is only possible to estimate the relative effects of a dummy variable. Although there are drawbacks, OB decomposition can be used broadly to implement the comprehensive analysis of inequality decomposition to identify the various factors for the change in inequality.

The other method is a priori approaches without regression such as factor-source decomposition and subgroup decomposition; these are also used to decompose the factors for the change in inequality. In this paper I focus on the subgroup-decomposition method shown below.

2.2 The inequality decomposition method by Mookherjee and Shorrocks

Subgroup decomposition can decompose inequality measures such as the Theil index, mean log deviation, and log variance into within- and between-group components based on workers’ attributes. As Mookherjee and Shorrocks (1982) used the mean log deviation (MLD), which has the desirable additive decomposability properties among subgroups, this paper also uses the MLD.

The MLD is defined as follows:

\[
MLD = \frac{1}{n} \sum_{i=1}^{m} \sum_{a=1}^{n_i} \ln \left( \frac{\mu}{x_{ia}} \right),
\]

where \( \mu \) is the overall mean wage and \( x_{ia} \) is the wage of worker \( a \) belonging to subgroup \( i \) composed of \( m \) groups. \( n \) represents the number of overall workers and \( n_i \) represents the number of workers in subgroup \( i \). The MLD is the average value of the log deviation width of the wages of each worker from the mean wage. If the MLD is large, it implies that the wage inequality is large.

Equation (1) decomposes the two components as follows: \(^5\)

\[
MLD = \sum_{i=1}^{m} \frac{n_i}{n} MLD_i + \sum_{i=1}^{m} \frac{n_i}{n} \ln \left( \frac{\mu}{\mu_i} \right),
\]

where \( \mu_i \) is the mean wage of group \( i \). The first term of equation (2) is a within-group component (the weighted sum of the inequalities within each subgroup). The second term

\(^5\)Generalized entropy indices that have additive decomposability, such as the Theil index, are able to decompose similarly.
is the between-group component, which reflects the inequality contribution due solely to differences in the subgroup means. Therefore, the total inequality equals the sum of these two contributions.

Following Mookherjee and Shorrocks (1982), the change in MLD between the two years, $t$ and $t + 1$, can be written as

$$
\Delta \text{MLD} = \text{MLD}(t + 1) - \text{MLD}(t) \\
\approx \sum_i \bar{s}_i \Delta MLD_i + \sum_i MLD_i \Delta s_i - \sum_i \bar{\lambda}_i \Delta s_i - \sum_i \bar{s}_i \Delta \ln \lambda_i \tag{3}
$$

$s_i \equiv \frac{n_i}{n}$ : workers’ share of group $i$

$MLD_i$ : MLD of group $i$

$\lambda_i \equiv \frac{w_i}{\bar{w}}$ : group $i$’s mean wage relative to the overall mean

$\theta_i \equiv s_i \lambda_i$ : group $i$’s wage share of the total wage of all workers

$\Delta$ is the difference operator between the two years $t$ and $t + 1$. A bar over the variables indicates the average of the base and following period values ($\bar{s}_i = \frac{s(t) + s(t+1)}{2}$). Overall inequality changes can be decomposed into (1) within-group inequality changes (term $A$), (2) changes resulting from changes in the composition of workers (terms $B$ and $C$) and (3) changes resulting from changes in the relative wages of different groups (term $D$: between-group inequality changes). In fact, term $B$ indicates the compositional effect caused by the change in the within-group component, while term $C$ indicates the compositional effect caused by the change in the between-group component in equation (3).

Many previous studies used equations (2) and (3) to analyze the factors for income and wage inequality changes (Mookherjee and Shorrocks 1982, Jenkins 1995, Oshio 2006, Oishi 2006, Yamaguchi 2011). They focused on only one factor, such as population aging. Therefore, their results omit the effects of other factors on inequality change, and thus, error analysis may occur, as shown in section 3. To resolve the problem, I extend the single subgroup decomposition method to a multiple one, as shown mathematically in the next subsection.

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6 Although this method is useful for identifying a specific single factor behind the change in inequality, it cannot perform simultaneous decomposition into multiple factors. When we analyze multiple factors for the change in inequality using subgroup-decomposition method, we must independently repeatedly decompose inequality measures into a single factor.

7 Although Mookherjee and Shorrocks (1982) suggested that multiple subgroup decomposition is feasible, they did not show a mathematical background or an empirical application in their paper.
2.3 Extending the inequality decomposition method

In equation (2), subgroup $i$ is divisible by another subgroup $k$: therefore, equation (2) can be written as follows:

$$MLD = \sum_{i=1}^{m} \frac{n_i}{n} \left[ \frac{\sum_{k} n_{ik}}{n_i} MLD_{ik} + \sum_{k} \frac{n_{ik}}{n_i} \ln \left( \frac{\mu_i}{\mu_{ik}} \right) \right] + \sum_{i=1}^{m} \frac{n_i}{n} \ln \left( \frac{\mu_i}{\mu_i} \right),$$

$$= \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} MLD_{ik} + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu_i}{\mu_{ik}} \right) + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu_i}{\mu_i} \right),$$

$$= \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} MLD_{ik} + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu_i}{\mu_{ik}} \right) + \ln \left( \frac{\mu_i}{\mu_i} \right),$$

$$= \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} MLD_{ik} + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu_i}{\mu_{ik}} \right),$$

where $MLD_{ik}$ is the MLD of subgroup $ik$ which is divided by subgroup $i$ into subgroup $k$; $\mu_{ik}$ represents the mean wage of group $ik$; and $n_{ik}$ represents the number of workers in group $ik$. I have used $MLD_i = \sum_k \frac{n_{ik}}{n_i} MLD_{ik} + \sum_k \frac{n_{ik}}{n_i} \ln \left( \frac{\mu_i}{\mu_{ik}} \right)$ to derive the first equation in (4) and $\frac{n_i}{n} \ln \left( \frac{\mu_i}{\mu_i} \right) = \sum_k \frac{n_{ik}}{n_i} \ln \left( \frac{\mu_i}{\mu_{ik}} \right)$ for the second equation. Equation (4) represents the decomposition of worker’s two attributes (e.g., age and education qualifications) into within-group and between-group components. As in (3), the change in the MLD between the two years, $t$ and $t+1$, can be also derived from (4), which can identify factors for the change in the MLD under controlling two factors.

Further, considering subgroup $l$ in subgroup $k$, equation (4) can be written as follows:

$$MLD = \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} MLD_{ikl} + \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} \ln \left( \frac{\mu_i}{\mu_{ikl}} \right),$$

where $MLD_{ikl}$ is the MLD of the subgroup $ikl$, which is divided by subgroup $ik$ into group $l$; $\mu_{ikl}$ represents the mean wage of subgroup $ikl$; and $n_{ikl}$ represents the number of workers in subgroup $ikl$.

Equation (5) indicates the decomposition of worker’s three attributes into within-group and between-group components. Repeating the substitution similarly with equations (4) and (5), the division of a subgroup can be increased without limit. However, it becomes impossible to perform the decomposition analysis, when the division of a subgroup is increased too much and a sufficient sample cannot be secured for actual analysis.\(^8\)

Similar to the derivation of equation (3), equation (5) can be written as follows:

$$\Delta MLD = MLD(t+1) - MLD(t),$$

\(^8\)When practitioner controls for some additional characteristics, one would need a discretisation of variables which might reasonably be considered as continuous (e.g., age).
\[
\begin{align*}
\sum_i \sum_k \sum_l \hat{s}_{ikl} \Delta MLD_{ikl} & + \sum_i \sum_k \sum_l MLD_{ikl} \Delta s_{ijl} - \sum_i \sum_k \sum_l \ln \lambda_{ijkl} \Delta s_{ijkl} - \sum_i \sum_k \sum_l \hat{s}_{ikl} \Delta \ln \lambda_{ikl}, \\
\approx \sum_i \sum_k \sum_l \hat{s}_{ikl} \Delta MLD_{ikl} + \sum_i \sum_k \sum_l MLD_{ikl} \Delta s_{ikl} & + \sum_i \sum_k \sum_l \left[ \lambda_{ikl} - \ln \lambda_{ikl} \right] \Delta s_{ikl} + \sum_i \sum_k \sum_l \left[ \hat{\theta}_{ikl} - \hat{s}_{ikl} \right] \Delta \ln \mu_{ikl}.
\end{align*}
\]

(6)

\(s_{ikl} \equiv \frac{n_{ikl}}{n} : \) workers’ share of group \(ikl\)

\(MLD_{ikl} : \) MLD of group \(ikl\)

\(\lambda_{ikl} \equiv \frac{n_{ikl}}{n} : \) group \(ikl\)’s mean wage relative to the overall mean

\(\theta_{ikl} \equiv s_{ikl} \lambda_{ikl} : \) group \(ikl\)’s wage share of the total wage of all workers

Overall inequality changes between periods \(t\) and \(t+1\) in (6) consist of within-group inequality changes (term A), changes resulting from changes in the composition of workers (terms B and C) and between-group inequality changes (term D). This deviation of multiple-factors decomposition method can be applied to other indices of inequality measurement such as the Thiel index and logarithmic variance, which have additive decomposability properties.

As stated above, an advantage of this proposed method is that it can simultaneously decompose two or more factors for changes in inequality without regression. This method is useful for analyzing the impact of population compositional effects such as education age and skills on the change in inequality measures. However, the drawback of this method is that it cannot analyze the factors for changes in income distribution or quantiles.

In the next section, an empirical application is performed to analyze the factors for wage inequality and its change in Japan in order to show the strengths of this method.

### 3 Empirical application: The Japanese wage inequality from 1992 to 2002

#### 3.1 Data

The data used for this study are micro data from the Employment Status Survey (ESS), which is conducted every five years by the Statistics Bureau of the Ministry of Internal Affairs and Communications in Japan. The ESS is a large-scale cross sectional
survey covering the complete population,¹ that was conducted on household members 15 years old or more in approximately 440,000 households dwelling in sampled units. This paper uses eighty percent of the original sample that was randomly chosen as anonymous resampled data: this data excludes households of eight or more persons, households with three or more persons of the same age who have less than 15 years of household membership, and the households of a specific institution from 1992 to 2002.¹⁰ The ESS obtains detailed records of the properties of workers: it includes income, sex, education qualifications, age, type, employment status, and firm size, work hours per week, and working days per year as of October 1 of each survey year. In the ESS, respondents answered queries about their annual income or wage by choosing from among given income ranges.¹¹ Therefore, the class value of range of annual wage is used to calculate inequality measures below.¹²

I analyze the wage inequality among full-time workers employed by firms with more than five persons excluding self-employed. Full-time workers here are those who work more than 35 hours per week and more than 200 days per year.

Insert Table 1 Wage inequality among full-time workers

Table 1 shows the transition of the MLD, the Gini coefficient, and the Theil index. It shows that each inequality measure has the same trends. Among male workers, wage inequality had a constant or slightly decreasing trend from 1992 to 1997, but increased from 1997 to 2002.¹³ Among female workers, wage inequality increased both from 1992 to 1997 and from 1997 to 2002.

Insert Fig. 1 Percentage of male full-time workers classified by annual wage range

Next, I examine how the income distribution changes. Figure 1 shows the proportion of

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¹The following persons however were excluded from the enumeration. Foreign diplomatic corps or consular staff and foreign military personnel or civilians, and their dependents. Persons dwelling in camps or ships of the Self-Defense Forces. Persons serving sentences in prisons or detention houses and inmates of reformatory institutions or women's guidance homes.

¹⁰I obtained this data from the Research Centre for Information and Statistics of Social Science, Institute of Economic Research, Hitotsubashi University.

¹¹Income or wage here mean the annual income or wage (inclusive of tax) that workers ordinarily earn from their main jobs during the past year, i.e., wages, salaries, various allowances, bonuses and the like.

¹²Since ESS provides not the precise data of working hours and days numbers but the working hours and days ranges chosen by respondents, it is difficult to work out the precise hourly wage.

¹³The inequality trend here is similar to the results of the log variance trend among permanent ordinary workers in Kambayashi et al. (2008).
the male full-time workers ranked based on 14 income brackets. The income ranges are, in tens of thousands of yen, 0–50, 50–99, 100–149, 150–199, 200–249, 250–299, 300–399, 400–499, 500–699, 700–999, 1000–1499, and 1500 and above. Figure 1 is depicted with a class mark for each range. From 1992 to 1997, the proportion of male full-time workers earning less than 3 million yen decreased slightly. The proportion of workers earning 3–5 million yen also decreased slightly, while the proportion of workers earning 6–15 million yen increased greatly. The wage inequality became constant or decreased slightly as the proportion of workers in the lower-income groups decreased, but those in higher-income groups increased from 1992 to 1997. In contrast, from 1997 to 2002, the proportion of workers earning less than 3 million yen increased slightly, the proportion of workers earning 4–7 million yen decreased greatly, and the proportion of workers earning more than 7 million yen decreased slightly. The wage inequality increased as the proportion of workers in the lower-income groups increased, and the proportion of workers in the middle-income groups decreased from 1997 to 2002.

Insert Fig. 2 Percentage of female workers classified by annual wage range

Figure 2 shows the proportion of female full-time workers classified by annual wage range. From 1992 to 1997, the proportion of workers earning less than 2.5 million yen decreased, whereas the proportion of workers earning 3–10 million yen increased substantially. Both the reduction in the proportion of low-income workers and the rise in the proportions of middle- and high-income workers are more noticeable than those of male workers. From 1997 to 2002, the proportion of workers earning 2–4 million yen decreased slightly, while the proportion of workers earning 4–10 million yen increased. The proportion of full-time male workers in the middle- and high-income groups decreased greatly, but that of females increased notably.

3.2 Wage inequality decomposition by a single subgroup

I focus on male full-time workers in the analysis of inequality decomposition. “Age” in Table 2 shows the results of the single sub-group decomposition calculated by (2) and (3). In the analysis, workers are divided into 12 groups on the basis of their age. Table 2 shows that the MLD among male full-time workers decreased by 1.5 from 1992 to 1997, but increased by 4.4 from 1997 to 2002.

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14For the annual wage ranges of the original data, there are 14 classifications in both 1992 and 1997 and 17 in 2002. To properly examine wage inequality over this period, the annual wage classification in each year is unified into 14 classifications.

15There were 12 classifications of age groups, which included five-year interbals from 15–19 years old through 65–69 years old, plus 70 and over.
Insert Table 2 MLD decomposition of age and age, education and firm size among full-time workers

Table 2 shows that the change in MLD from 1992 to 1997 is decomposed into the increasing between-group inequality (term D), the decreasing within-group inequality (term A), and the negative compositional effects (terms B and C). For 1997 to 2002, the increase in overall wage inequality is decomposed into the strongly increasing within-group inequality, the increasing between-group inequality, and the decreasing compositional effects. These results seem to imply that the between-age-group inequality of full-time workers became larger in those periods. However, these findings are inconsistent with previous research that shows male full-time workers narrowing wage differential between age groups. Mitani (2005) shows that the relative wages of both 40–49 and over 50 years old to 15–24 years old decreased both from 1985 to 1993 and from 1993 to 2000 when the composition change in age and education group are controlled. This contradiction is deemed to be caused by the implementation of only the age-subgroup decomposition, which does not control for any other factors, and which suggests that a single subgroup-decomposition method should be enriched by controlling for any other factors. In the next subsection, I show that the multiple subgroup-decomposition method is useful for resolving this problem.

3.3 Wage inequality decomposition by multiple subgroups

In Table 2, “age education and firm size” shows the results of the multiple subgroup-decomposition using equations (5) and (6). In the analysis, workers are divided into groups on the basis of their age (12 classifications), education (four classifications), and firm size where workers work (nine classifications). The results show that the decreasing within-group inequality (term A) and between-group inequality (term D) and the positive compositional effects (terms B and C) contribute to the change in overall wage inequality from 1992 to 1997. From 1997 to 2002, the strongly positive within-group inequality and weakly positive between-group inequality dominate the negative compositional effects. Even if we extend the single decomposition

16 Using microdata from the Basic Survey on Wage Structure, Hamaaki et al. (2010) construct an age-earnings profile of a median worker who remained with the same employer. They examine the time-series variation of its slope between 1989 and 2008 and find that the wage profile gradually became flatter in the 1990s, and thereafter, the profile appeared to have eventually kinked, that is, it was a nearly non-increasing wage in the latter half of career life in 2007–2008.

17 The educational background groups are elementary school or junior high school graduates (9 years of compulsory schooling), high school graduates (12 years of schooling), junior college or college of technology graduates (usually 14 years of schooling), and university and graduate school graduates (16 years or more of schooling).

18 There are nine classifications for firm size: 5–9 persons, 10–19 persons, 20–29 persons, 30–49 persons, 50–99 persons, 100–299 persons, 300–499 persons, 500–999 persons, and 1000 or more persons.
method to multiple one, the factors for the expanding overall inequality of full-time workers from 1997 to 2002 are mainly attributed to the increase in within-group inequality. The increase in within-group inequality for groups of identical age, education, and firm size may be caused by the changes in the wage system, such as the extensive introduction of performance pay and the weakening wage negotiation attitude of labor unions in the late 1990s.

I next move on to the analysis of the contradictory between-age-group inequality stated above. The multiple subgroup-decomposition method can identify the specific factor controlling the rest of the factors.\textsuperscript{19} Table 3 shows the effect of age on the overall inequality, by substructing the MLD decomposition results considering age, education, and firm size from the MLD decomposition results considering education and firm size. This method is very useful for identifying the specific effect. Appendix 1 includes a mathmatical background for this method. The right column in Table 3 shows the relative age effects: between-age-group inequality became lower and within-group inequality became larger both from 1992 to 1997 and from 1997 to 2002. These findings are consistent with previous research such as Mitani (2005). This result implies that the single subgroup-decomposition method has a limit that the decreasing effects of between-age-group inequality could not identified as shown in Table 2, and thus practitioners should use the multiple decomposition method when implementing the inequality decomposition.

Further, I extract the education effect in Table 4 in the same manner. Table 4 shows that the compositional change of education (Term C) has a positive effect on the between-group component. This implies that the increase in the relative supply of educated workers seemingly make the between-group component larger.\textsuperscript{20} In addition, except for compositional change, the between-education-group inequality has a negative effect on over-

\textsuperscript{19}Cowell and Jenkins (1995) develop the $R_B$ statistics which can measure the share of overall inequality accounted for by population characteristics. Where $R_B$ statistics is defined as the ratio of between-group component partitioning an group characteristic attributes to overall inequality. Bourguignon et al. (2008) observe the limitation of $R_B$ statistics: none of the decompositions control for any of the others. In addition, the share of total inequality attributed to that partition tells us nothing of whether it is the distribution of the characteristic or the structure of its returns that matters. The method of this paper can resolve the first limitation of $R_B$ proposed by Bourguignon et al. (2008).

\textsuperscript{20}Genda (1997) shows that the increase in the relative supply of the college graduates holds down the increase in the wage of college graduates in the early 1990s.
This implies the pure between-education-group inequality for male full-time workers may become lower in those periods as opposed to the increasing college premium in US caused by skill biased technical change. The decrease in between-education-group inequality may be subject to using the male full-time data in the analysis. Further analysis in the future using the data including part-time and temporary workers are left.

Table 5 shows the effect of firm size extracted. The compositional effects are not so much both from 1992 to 1997 and from 1997 to 2002. The between-firm-size-group inequality became much larger from 1997 to 2002 than from 1992 to 1997. This was maybe caused by that the stagnation and financial crisis in the late 1990s that forced smaller firms to be strict position than the other larger firms.

3.4 Contribution by each attributes to overall decomposition

As shown above, the multiple subgroup-decomposition method is useful for decomposing the overall inequality for analyzing the specific factors for inequality change under controlling the other factors. In addition, this method can decompose the overall inequality into each contribution of age group, education group, and firm-size group. Appendix 2 includes the mathematical background. Table 6 shows the contribution by each attribute division to the change in the overall inequality. The first row in Table 6 indicates the overall decomposition change which is the same as the lower part of the right column in Table 2. The second row and below indicate the contribution by each worker attribute to the change in the overall within-group inequality (term A), the overall compositional change of the within-group component (term B), the overall compositional change of the between-group component (term C) and the change in overall between-group inequality (term D). Note that the contribution by each group does not mean the effect of each attribute on the overall decomposition as shown in Tables 3 and 4, but rather the contribution of each group on the overall decomposition.22

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21Kambayashi et al. (2008) observe a large decline of the return to two- and four-year college education between 1989 and 2003. They surmise that this decline may be explained by the increase in the supply of educated workers.

22The contribution of age group, education group, and firm size group are respectively one-third of over all inequality, as shown in Appendix 2.
For example, the change in overall within-group inequality is 5.5 from 1997 to 2002 which is summed up in each contribution for term A. The largest contributions are from high school graduates (12 years of schooling), 0.8; firms with more than 1,000, 0.7; and those with at least 16 years of schooling, 0.6. Within-group inequality became larger among those in age groups between 30 and 59.

The education group is notable for compositional effects. The relative increase in the effect on between-group inequality (Term C) among highly educated groups (14 years or more of schooling) and the relative decrease in the effect among the groups with 12 years or fewer of schooling are profound. While the former effect dominates the latter from 1992 to 1997, their effects are reversed from 1997 to 2002.

4 Conclusion

This paper proposed the multiple inequality decomposition method to which is extended the single subgroup-decomposition method. The multiple subgroup-decomposition method is able to control and identify multiple factors for inequality and its change without any difficulty. This method is also able to decompose the overall inequality and its change into the contributions of subgroups. This method does not need to estimate the regression like the OB decomposition method, and therefore it does not incur the regression error caused by the endogeneity problem.

Using this method, this paper analyzed the factors for the change in wage inequality among full-time male workers in Japan from 1992 to 2002 based on a micro-level data set from the Employment Status Survey (ESS).

Wage inequality had a constant or slightly decreasing trend from 1992 to 1997, however, increased from 1997 to 2002. The decreasing trends of between-group component both from 1992 to 1997 and 1997 to 2002 were observed. These were due to the decreasing effect of between-age-group inequality and between-education-group inequality and increasing effect of between-firm-size-group inequality both from 1992 to 1997 and from 1997 to 2002, although the increasing effect by the increase in higher education seemingly made the between-education-group inequality larger. In regards to the method, this paper found that the decreasing effects of between-age-group inequality could not be identified by the single subgroup-decomposition, and thus, practitioners should use the multiple decomposition method when implementing the inequality decomposition precisely.

In addition, this paper observed that the factors for the expanding overall inequality from 1997 to 2002 were mainly attributed to the increase in within-group inequality for groups of identical age, education, and firm size. The increase might be caused by the extensive introduction of performance pay in the late 1990s.

Some topics are left for future research. First, the multiple subgroup-decomposition method in this paper should be compared with the OB decomposition method using the same data to show how the regression error affects the inequality decomposition. Second,
the Japanese labor market underwent significant change in the late 1990s. The erosion of the lifetime employment system, introduction of paformance-pay and the increase in the part-time and temporary workers are more likely to have caused the changes in wage distribution and increased wage inequality.\textsuperscript{23} Strict empirical analysis of the relation between increasing wage inequality and institutional change or technical change are left for future research.

**References**


\textsuperscript{23}Moriguchi (2010) showed using time series regression analysis that marginal income tax rates, corporate performance, female labor force participation, and labor disputes are important determinants of top wage income shares in post-World War II Japan. Using the dataset of the Employment Status Survey (ESS) including part-time and temporary workers, Ohta (2005) showed that the increase in wage inequality among young workers was caused by the increase in non-regular employees such as part-time, short-time contract and temporary workers. By using the data from Special Survey of Labor Force Survey, Shinozaki (2006) showed that the expansion of the wage inequality between 1985 and 2005 could be attributed to the drastic increase in the percentage of non-regular employees.


5 Appendix 1. The effect of the additive subgroup

The effect of a specific factor on overall inequality can be identified by calculating the effect of the additive subgroup on the overall inequality and its change. Appendix 1 derives the effect of the additive subgroup. Equation (5) can be derived from (4) as follows.

\[
MLD = \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} MLD_{ik} + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu_i}{\mu_{ik}} \right), \quad (4)
\]
\[
\begin{aligned}
&= m \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \left[ \sum_{l} \frac{n_{ikl}}{n_{ik}} MLD_{ikl} + \sum_{l} \frac{n_{ikl}}{n_{ik}} \ln \left( \frac{\mu_{ik}}{\mu_{ikl}} \right) \right] + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu}{\mu_{ik}} \right), \\
&= m \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} MLD_{ikl} + \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} \ln \left( \frac{\mu_{ik}}{\mu_{ikl}} \right) + \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} \ln \left( \frac{\mu}{\mu_{ik}} \right), \\
&= m \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} MLD_{ikl} + \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} \ln \left( \frac{\mu}{\mu_{ikl}} \right). 
\end{aligned}
\]  

(7)

Subtracting (7) from (4) yields the following equation (8):

\[
\begin{aligned}
&= \left[ \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} MLD_{ikl} - \sum_{i=1}^{m} \sum_{k} \frac{n_{ik}}{n} MLD_{ik} \right] + \sum_{i=1}^{m} \sum_{k} \sum_{l} \frac{n_{ikl}}{n} \ln \left( \frac{\mu_{ikl}}{\mu_{ik}} \right). 
\end{aligned}
\]  

(8)

Equation (8) consists of the first parenthetic term and the second term. The first term is the decreasing effect by controlling the additional subgroup \( l \) on the within-group component, and the second term is the increasing effect on the between-group component. The sum of these effects, as shown in the upper partition of the right columns in Tables 3 and 4, becomes zero because (5) is equal to (7). The second term in (8) represents the differential between \( \mu_{ik} \) and \( \mu_{ikl} \) which implies a between-group inequality among subgroup \( l \). Cowell and Jenkins (1995) proposed the \( R_B \) statistics which can calculate how much inequality can be explained. Shourocks (2013) proposed the Shapley decomposition method for inequality which can assign contributions to the explanatory factors. However, these method cannot control the other factors precisely when they extract the contribution of a factor to overall inequality.\(^{24}\) Following the idea of \( R_B \) statistics, the second term in (8) can be regarded as the contribution of the effect of worker’s attribute \( l \) to the overall inequality under controlling other factors such as worker’s attribute \( i \) and \( k \).

The change in equation (8) between periods \( t \) and \( t+1 \) can be decomposed into within-group inequality changes (term A), changes resulting from changes in the composition of workers (terms B and C) and between-group inequality changes (term D); this is the same as the change in overall MLD between periods \( t \) and \( t+1 \) in (6). Subtracting the time variation of (6) from the time variation of (4), I can obtain the time variation in equation (8) as follows:

\[
\begin{aligned}
&\left[ \sum_{i} \sum_{k} \sum_{l} \bar{s}_{ikl} \Delta MLD_{ikl} - \sum_{i} \sum_{k} \bar{s}_{ik} \Delta MLD_{ik} \right] + \left[ \sum_{i} \sum_{k} \sum_{l} MLD_{ikl} \Delta s_{ikl} - \sum_{i} \sum_{k} MLD_{ik} \Delta s_{ik} \right] \\
&\text{term A} \quad \text{term B}
\end{aligned}
\]

\(^{24}\)Shapley decomposition can resolve “path dependence” problem by considering the all possible elimination sequences and its expected value. However, this procedure entails the error partly because some possible elimination sequences include the effects without controlling the other factors.
Term $A$ represents the time variation of within-group inequality, terms $B$ and $C$ represent the time variation of compositional change, and term $D$ represents the time variation of between-group inequality when considering the additive subgroup $l$. That is, the decomposition in equation (9) implies the relative effects of innovating the additive subgroup $l$ on the time variation of equation (6).

6 Appendix 2. Contributions by each attribute to overall inequality change

Appendix 2. shows how to calculate contributions by each attribute to overall inequality change. The term $A$, $\sum_i \sum_k \sum_l \bar{s}_{ikl} \Delta MLD_{ikl}$ in equation (6) can be rewritten as follows:

$$\text{Term } A = \frac{\sum_i \sum_k \sum_l \bar{s}_{ikl} \Delta MLD_{ikl} + \sum_i \sum_k \sum_l \bar{s}_{ikl} \Delta MLD_{ikl} + \sum_i \sum_k \sum_l \bar{s}_{ikl} \Delta MLD_{ikl}}{3},$$

where $\Delta \text{Within}_i = \sum_k \sum_l \bar{s}_{ikl} \Delta MLD_{ikl}$ which represents the change in within-group inequality among subgroup $i$. $\Delta \text{Within}_k$ and $\Delta \text{Within}_l$ are defined in a similar fashion. Equation (10) indicates the contributions by subgroup $i$, $j$, and $k$ to the change in within-group inequality.

In a similar way, the terms $B$, $C$ and $D$ in Equation (6) can be rewritten as follows:

$$\text{Term } B = \frac{\sum_i \Delta s_i^{\text{within}} + \sum_k \Delta s_k^{\text{within}} + \sum_l \Delta s_l^{\text{within}}}{3},$$

$$\text{Term } C = \frac{\sum_i \Delta s_i^{\text{between}} + \sum_k \Delta s_k^{\text{between}} + \sum_l \Delta s_l^{\text{between}}}{3},$$

$$\text{Term } D = \frac{\sum_i \Delta \text{Between}_i + \sum_k \Delta \text{Between}_k + \sum_l \Delta \text{Between}_l}{3},$$

where $\Delta s_i^{\text{within}} = \sum_k \sum_l MLD_{ikl} \Delta s_{ikl}$, which represents the compositional effect of the within-group component in subgroup $i$, $\Delta s_i^{\text{between}} = \sum_k \sum_l \lambda_{ikl} - \ln \lambda_{ikl} \Delta s_{ikl}$, which represents the compositional effect of the between-group component in subgroup $i$, and
\[ \Delta \text{Between}_i \equiv \sum_k \sum_l \left[ \bar{\theta}_{ikl} - \bar{s}_{ikl} \right] \Delta \ln \bar{\mu}_{ikl}, \] which represents the change in the between-group inequality in subgroup \( i \).

Equations (11) and (12) indicate the contributions by subgroup \( i, k, \) and \( l \) to the compositional changes in the within-group and between-group components, respectively. Equation (13) indicates the contributions by subgroups \( i, k, \) and \( l \) to the change in the between-group inequality.

\[ ^{25}\text{The compositional and between-group inequality change effects among subgroups } k \text{ and } l \text{ are defined in a similar fashion.} \]
Fig 1: Percentage of male workers classified by annual wage range
Fig 2: Percentage of female workers classified by annual wage range
Table 1: Wage inequality of full-time workers in firms with 5 or more employees

<table>
<thead>
<tr>
<th>Year</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean log deviation (MLD)</td>
<td>0.135</td>
<td>0.134</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>0.283</td>
<td>0.281</td>
</tr>
<tr>
<td>Theil index</td>
<td>0.135</td>
<td>0.133</td>
</tr>
</tbody>
</table>
Table 2: MLD decomposition of age and age, education, and firm size among full-time workers

<table>
<thead>
<tr>
<th>Year</th>
<th>age</th>
<th>age education, and firm size</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>135.2</td>
<td>133.7</td>
</tr>
<tr>
<td>Within-group component</td>
<td>90.7</td>
<td>88.8</td>
</tr>
<tr>
<td>Between-group component</td>
<td>44.4</td>
<td>44.9</td>
</tr>
</tbody>
</table>

Change in aggregate inequality from five years ago

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term A (within-group inequalities)</td>
<td>-3.2</td>
<td>8.3</td>
<td>-3.2</td>
<td>5.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term B (composition)</td>
<td>1.3</td>
<td>0.6</td>
<td>0.8</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term C (composition)</td>
<td>-3.1</td>
<td>-6.1</td>
<td>1.5</td>
<td>-2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term D (between-group inequalities)</td>
<td>3.6</td>
<td>1.6</td>
<td>-2.4</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers are in thousands. The value of each term of decomposition includes the error. The values of the upper partition are calculated by equations (2) and (5). The values of the lower partition are calculated by equations (3) and (6).
Table 3: Age effect

<table>
<thead>
<tr>
<th>Year</th>
<th>education and firm size</th>
<th>the effect of age</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>135.2</td>
<td>133.7</td>
</tr>
<tr>
<td>Within-group component</td>
<td>108.5</td>
<td>106.1</td>
</tr>
<tr>
<td>Between-group component</td>
<td>26.7</td>
<td>27.6</td>
</tr>
</tbody>
</table>

Change in aggregate inequality from five years ago: -1.5, 4.4

Term A (within-group inequalities): -3.2, -1.0, 1.9, 6.4
Term B (composition): 0.8, 0.8, 0.0, -0.4
Term C (composition): 0.2, -0.4, 1.3, -1.8
Term D (between-group inequalities): 0.7, 4.9, -3.1, -3.9

Notes: Numbers are in thousands. The value of each term of decomposition includes the error. “Education and firm size” indicates the MLD decomposition results considering education and firm size. “The effect of age” indicates the calculated difference between MLD decomposition results considering age, education, and firm size and those considering education and firm size. The decomposition results of the effect of age includes the error from deduction.
### Table 4. Education effect

<table>
<thead>
<tr>
<th>Year</th>
<th>age and firm size</th>
<th>the effect of education</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>135.2</td>
<td>133.7</td>
</tr>
<tr>
<td>Within-group component</td>
<td>73.0</td>
<td>71.2</td>
</tr>
<tr>
<td>Between-group component</td>
<td>62.2</td>
<td>62.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>1992</th>
<th>1997</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term A (within-group inequalities)</td>
<td>-2.7</td>
<td>5.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Term B (composition)</td>
<td>0.9</td>
<td>0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Term C (composition)</td>
<td>-2.1</td>
<td>-5.0</td>
<td>3.6</td>
</tr>
<tr>
<td>Term D (between-group inequalities)</td>
<td>2.5</td>
<td>3.6</td>
<td>-4.9</td>
</tr>
</tbody>
</table>

**Notes:** Numbers are in thousands. The value of each term of decomposition includes the error.  
"Age and firm size" indicates the MLD decomposition results considering education and firm size.  
"The effect of education" indicates the calculated difference between MLD decomposition results considering age, education, and firm size and those considering age and firm size.  
The decomposition results of the effect of education include the error from deduction.
Table 5: The effect of firm size

<table>
<thead>
<tr>
<th>Year</th>
<th>age and education</th>
<th>the effect of firm size</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>135.2</td>
<td>133.7</td>
</tr>
<tr>
<td>Within-group component</td>
<td>74.1</td>
<td>73.8</td>
</tr>
<tr>
<td>Between-group component</td>
<td>61.1</td>
<td>59.9</td>
</tr>
<tr>
<td>Change in aggregate inequality from five years ago</td>
<td>-1.5</td>
<td>4.4</td>
</tr>
<tr>
<td>Term A (within-group inequalities)</td>
<td>-1.1</td>
<td>8.3</td>
</tr>
<tr>
<td>Term B (composition)</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Term C (composition)</td>
<td>1.4</td>
<td>-2.5</td>
</tr>
<tr>
<td>Term D (between-group inequalities)</td>
<td>-2.6</td>
<td>-1.8</td>
</tr>
</tbody>
</table>

Notes: Numbers are in thousands. The value of each term of decomposition includes the error. "Age and education" indicates the MLD decomposition results considering age and education. "The effect of firm size" indicates the calculated difference between MLD decomposition results considering age, education, and firm size and those considering age and education. The decomposition results of the effect of education include the error from deduction.
Table 6: The contributions by each attribute of workers to the change in overall MLD

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>term A</td>
<td>term B</td>
</tr>
<tr>
<td>9 years of compulsory schooling</td>
<td>-0.2</td>
<td>-0.8</td>
</tr>
<tr>
<td>12 years of schooling</td>
<td>-0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>14 years of schooling</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>16 years or more of schooling</td>
<td>0.1</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>15–19</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20–24</td>
<td>0.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>25–29</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>30–34</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>35–39</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>40–44</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>45–49</td>
<td>0.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>50–54</td>
<td>-0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>55–59</td>
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</tr>
<tr>
<td>60–64</td>
<td>-0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>65–69</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>70+</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5–9 persons</td>
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</tr>
<tr>
<td>10–19 persons</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>20–29 persons</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>30–49 persons</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>50–99 persons</td>
<td>0.1</td>
<td>0.1</td>
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<tr>
<td>100–299 persons</td>
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<td>0.1</td>
</tr>
<tr>
<td>300–499 persons</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>500–999 persons</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>1,000 persons+</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>